When and How Much Does a Peg Increase Trade? The Role of Trade Costs and Import Demand Elasticity under Monetary Uncertainty

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Abstract

This paper extends stochastic research in new open-economy macroeconomics (NOEM) to study the effects of the exchange-rate regime on international trade in a more realistic, yet rigorous, analytical set-up. We essentially incorporate "iceberg" costs, inducing home bias, into a unified framework which nests trade between countries that produce similar vs. different composite goods. Our main result is that given (some degree of) producer's currency pricing with symmetry in structure and money shock distributions as the only source of uncertainty, a fixed exchange rate slightly reduces expected trade, measured as a share in GDP, relative to a float under *elastic* import demand, i.e. when countries' output mixes are similar; *inelastic* import demand, possible under the same taste for diversity but far less substitutable national outputs arising in our model from differences in endowments although not in technological labor input requirements, reverses this conclusion. What a peg can achieve in any of these cases is trade stabilization (across states of nature). It would be greater for (symmetric) nations which (i) have a larger proportion of producer's currency pricing in their trade, (ii) are exposed to higher monetary uncertainty, (iii) produce less substitutable output mixes and (iv) are located closer to one another or apply weaker bilateral trade restrictions.

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1 Introduction

The voluminous literature that has directly or indirectly asked the question whether peg vs. float matters for trade and/or welfare has not arrived yet at a satisfactory answer. A fixed exchange rate has often been claimed to substantially increase trade, mostly on empirical grounds as, for instance, in Rose (1999). Bacchetta-van Wincoop (2000) and Mihailov (2003) have, however, concluded in related theoretical work that this should not necessarily be the case. The objective of the present paper is thus to examine further the effects of the exchange-rate regime on trade prices and flows in a careful manner, by also looking into some of their key non-monetary determinants. Building on the stochastic new open-economy macroeconomics (NOEM) set-up in Mihailov (2003) which parallels a consumer's currency pricing (CCP) model version to a producer's currency pricing (PCP) one, we first incorporate broadly interpreted "transport" costs into a baseline NOEM model. We then study the role such a market friction plays in an extended and unified framework that allows to nest alternative types of trade in the simplest analytical way.

The relevant literature, classic as well as NOEM, has usually modelled in separation either trade of differentiated brands belonging to the same homogeneous product[1] or trade arising from complete specialization in the production of just one national good-type.[2] A major import of the present paper is that it embeds trade in similar vs. different output mixes within a *common* theoretical framework, at the same time taking an explicit account of impediments to cross-border transactions. Our unified approach becomes feasible, it is true, at the cost of a highly stylized environment, by essentially attributing the primary cause of the international exchange of goods to identical tastes for diversity and not to Ricardian comparative advantage in productivity. Nevertheless, a micro-founded general-equilibrium analysis of the effects of the exchange-rate regime on *both* types of trade in the presence of *barriers*, in addition to the monetary uncertainty and the CCP vs. PCP nominal rigidity of our initial set-up, appears justified as an insightful first shortcut.

Our results basically show that, although robust to some of the generalizations introduced, the frictionless, single substitutability NOEM set-up, in particular Bacchetta and van Wincoop (1998, 2000 a, b) and Mihailov (2003), is rendered better suited to explain the role the exchange-rate regime plays in influencing international trade when extended in the ways we propose here. Once trade costs and distinct cross-country substitutability are integrated into

[1] As in Obstfeld-Rogoff (1995, 1998, 2000, 2001) models, to quote just the earliest NOEM examples.

[2] As in Corsetti-Pesenti (1997, 2001 a, 2001 b, 2002) extensions (under unit substitutability across national good-types) of the original Obstfeld-Rogoff framework.

it, equilibrium outcomes tend to duly emphasize – beyond money shocks and invoicing conventions – the preponderance in trade determination of these two deeper "fundamentals". In a preview of our principal findings, we could say that, first, *PPP does not hold* anymore under PCP, with even symmetric iceberg losses of the output shipped abroad. Second, national trade shares in GDP under production of similar goods drastically fall in the presence of cross-border barriers to their exchange. The reason is that such obstacles induce a *home bias in consumption* in the optimal behavior of agents with identical tastes, a relatively novel feature within NOEM. This bias is, furthermore, considerably mitigated to more empirically relevant levels under production of different output mixes under moderately elastic import demand. But the most important result the paper derives is that, unlike in Mihailov (2003), the exchange-rate regime affects under PCP *expected* trade-to-output ratios, in a way depending on the interaction of trade costs with the degree of substitutability between the nationally-produced composites: under elastic demand of similar products a peg slightly reduces expected trade relative to a float given the same symmetric distribution of money shocks, in both economies and, hence, for the world as a whole; under inelastic demand because of complete specialization in two different equally-valued good-types, a peg slightly reduces expected trade relative to a float. But this effect of the exchange-rate regime on *costly* trade under monetary uncertainty is not quantitatively significant, as also found in related NOEM literature. Another new point from our analysis is that non-monetary factors such as transport or tariff frictions and the substitutability of output mixes also determine, via the consumption bias, the variability of trade-to-output. As to the trade stabilization a peg can achieve under (some) PCP, a contribution of the present study is to clarify that its extent would be greater for countries or currency blocs which produce less substitutable good-types for meaningful costs of exchanging them and are located closer to one another or apply weaker restrictions in their bilateral trade.

The paper is further down organized as follows. Section 2 outlines our *extended* stochastic NOEM model of exchange rate and trade determination and highlights the differences in its initial assumptions under CCP vs. PCP. The third section studies under symmetry how *trade costs* and distinct *type* and *brand* consumption substitutabilities affect international relative prices and, consequently, agents' optimization and the key equilibrium relationships across our alternative invoicing. Section 4 then focuses on the effects of the exchange-rate regime on both the expected level and the variability of trade-to-output ratios, whereas the fifth section clarifies the role played by their real determinants. Section 6 concludes and appendices A and B contain, respectively, a detailed derivation of optimization and equilibrium results and the proofs of propositions.

2 The Extended Model

In this section, we first briefly outline the set-up in Mihailov (2003). We then explain how our two extensions here, the iceberg costs friction and the distinct cross-country substitutability, have been analytically integrated within it.

2.1 Our Baseline

General Environment Our stochastic economy exists in a single period[3] and is made up of two countries, *H(ome)* and *F(oreign)*, assumed of equal size. A continuum of differentiated *brands*, each produced and sold by a single monopolistically competitive firm, is available for consumption. Brands as well as their producers are indexed by i in H and i^* in F. In the version of the model we focus further down, all brands belong to two national good-*types* produced under complete specialization due to endowment differences. This more general version reduces to the case of a world economy producing varieties of a single good when substitutabilities across brands and types are not distinguished, as in Mihailov (2003). Firms in Home are uniformly distributed on $[0, 1]$ and those in Foreign on $(1, 2]$. We assume sticky prices motivated by menu costs. Monopolistic competition enables each firm to optimally choose the price(s) at which it sells its product. Prices are set in advance, i.e. in our *ex-ante* state 0 (*before* uncertainty has been resolved), and remain valid for just one period, i.e. for the *ex-post* state $s \in S$ we consider (*after* money shocks have been observed).

Governments and Shocks In each country, there is a government whose only role is to proportionally transfer cash denominated in national currency to all domestic households in a *random* way.[4] We interpret such a money supply behavior as a *flexible* exchange-rate system and model it in terms of stochastic money-stock *growth rates*. Moreover, we restrict it to be *jointly symmetric*, in the following sense. For $\forall s \in S$, μ_s and μ_s^* are, respectively, H-money stock and F-money stock *net* rates of growth, having the same means and variances. For the sake of symmetry, *ex-ante* (state 0) national money holdings of the representative households in Home and Foreign are assumed *identical* in terms of units of each country's currency:[5] $M_0 = M_0^*$. The *ex-post* (state s) cash balances, i.e. the domestic-currency budgets with which Home and Foreign households dispose for transactions purposes in the realized state of nature $s \in S$, are then respectively given by $M_s \equiv M_0 + \mu_s M_0 = (1 + \mu_s) M_0$ and $M_s^* \equiv M_0^* + \mu_s^* M_0^* = (1 + \mu_s^*) M_0^*$.

[3] Extension to sequential dynamics is straightforward: it will only violate ex-ante symmetry right after the first period and thus require recursive simulation. However, since the relevant measure of variables under uncertainty is their *expected* level, with which we are concerned here, simulating and summing over a sufficiently large number of periods will essentially replicate the analytically derived and simulated results over multiple states of nature we report further down.

[4] Seigniorage is then repaid in a lump-sum fashion, as is standard in the related literature.

[5] At an initial *equilibrium* exchange rate $S_0 = 1$, as will be discussed later.

The only difference between float vs. peg in terms of the (conditional) joint distribution (up to second moments, inclusive) of national money *growth* shocks (μ_s, μ_s^*) and, hence, of the resulting ex-post money *stocks* (M_s, M_s^*) thus arises from their *covariance* terms. It is imposed by the definition itself of a fixed vs. flexible exchange-rate regime: under (pure) *float*, the (conditional) correlation of national money stocks is 0; under (credible) *peg*, this (conditional) correlation is 1. In essence, our fixed exchange-rate version is thus isomorphic to a model where a monetary union or a single currency area is hit by just one, common money shock.

Households In H and F, there is a continuum of households assumed *identical.* The population in each of these economies is supposed constant and is normalized to 1. The representative household (in H as well as in F) likes diversity and *consumes all brands* on the interval $[0, 2]$. It also *supplies labor*, earning the equilibrium wage, and *owns an equal proportion of domestic firms*, receiving their profits (in the form of dividends).

The representative household in Home[6] maximizes its *ex-post* (state s) utility:

$$\underset{c_s, l_s}{Max} \quad u(c_s, l_s), \quad \forall s \in S. \tag{1}$$

Our utility function is assumed to be *well-behaved* (i.e. to exist, be continuous, twice differentiable and concave) and *separable.* l_s is (hours of) leisure and c_s is a *constant* elasticity of substitution (CES) real consumption index defined in the present paper by the following Dixit-Stiglitz (1977) aggregator:[7]

$$c_s \equiv \left\{ \left(\frac{1}{2}\right)^{\frac{1}{\nu}} \left[\left(\int_0^1 c_{i,s}^{\frac{\varphi-1}{\varphi}} di \right)^{\frac{\varphi}{\varphi-1}} \right]^{\frac{\nu-1}{\nu}} + \left(\frac{1}{2}\right)^{\frac{1}{\nu}} \left[\left(\int_1^2 c_{i^*,s}^{\frac{\varphi-1}{\varphi}} di^* \right)^{\frac{\varphi}{\varphi-1}} \right]^{\frac{\nu-1}{\nu}} \right\}^{\frac{\nu}{\nu-1}}. \tag{2}$$

In this representative agent economy, the aggregate constraints on (per-) household behavior coincide with those of the identical households. They are standard in NOEM but, for completeness, we briefly present them below.

Time Endowment Constraint The endowment of hours to the representative household (in Home) is normalized to 1 in each state,

[6] The notation in which the model is further on set out generally refers to Home, but for Foreign symmetric relationships hold unless otherwise stated and can usually be verified in the relevant appendices (on this particular point, see *Appendix A.1*).

[7] The analogous definition for the *Foreign* representative consumer can be verified in *Appendix A.1.* Note as well that at this point we introduce a first difference of the extended model considered further down with respect to the baseline we are summarizing (here, in subsection 2.1). This difference is that $\nu \neq \varphi$, whereas the corresponding simpler aggregator in Mihailov (2003) assumed $\nu \equiv \varphi$. More on that in subsection 2.3.

$$l_s + n_s \equiv 1, \quad \forall s \in S. \tag{3}$$

so that $n_s \equiv 1 - l_s$ is (Home) household's (hours of) labor (supply).

Cash-in-Advance (CiA) Constraint Households need to carry *cash* before going to the goods market. Moreover, we restrict them to hold and receive from their monetary authority only *domestic* currency. Thus (for Home)

$$\underbrace{c_s P_s}_{H \text{ national } \textit{expenditure} \text{ (in } H \text{ currency)}} \leq \underbrace{M_s}_{\text{available } \textit{cash} \text{ in } H \text{ (in } H \text{ currency)}}, \quad \forall s \in S. \tag{4}$$

National Money Market Equilibrium Since CiA constraints are *binding*[8] and there is no investment and government spending in the model, the nominal value of national output sold (for consumption) is equal to the total stock of money in each of the countries:

$$Y_s = M_s, \quad \forall s \in S. \tag{5}$$

Aggregate Budget Constraint = National Income Identity With a nominal *wage rate* of W_s and total *hours of work* amounting to $1-l_s$, the nominal *labor income* of the (Home) representative household is given by $W_s(1 - l_s)$. Nominal *dividends* from firm profits earned by this household are denoted by Π_s. *In equilibrium*, all income from the activity of firms is distributed to domestic households (this happens at the end of the one-period framework we consider):

$$\underbrace{\underbrace{W_s(1 - l_s)}_{\text{labor income}} + \underbrace{\Pi_s}_{\text{ownership income}}}_{H \text{ national (factor) } \textit{income} \text{ (in } H \text{ currency)}} \equiv \underbrace{Y_s}_{H \text{ national } \textit{output} \text{ (in } H \text{ currency)}}, \quad \forall s \in S. \tag{6}$$

First-Order Conditions The following *"compact" FONC* can be derived in a familiar way from the above-described constrained optimization problem for the H representative household:

$$W_s = \frac{u_{l,s}}{u_{c,s}} P_s, \quad \forall s \in S. \tag{7}$$

$u_{l,s}$ and $u_{c,s}$ in (7) are the marginal utilities of leisure and consumption, respectively, in the realized state s. The *real* wage rate is thus equal, in equilibrium, to the ratio of these marginal utilities.

[8] See Mihailov (2003).

Firms Production is effected by firms which are owned by *domestic* households only. We also abstract from an international stock market, as well as of risk-sharing issues in general. To simplify this initial analysis of trade in similar vs. different output mixes within a *unified* framework, we focus here on identical technologies in terms of labor input for producing a unit of output – although endowments may differ – common to all firms in Home (and symmetrically in Foreign):

$$y_s = n_s = 1 - l_s. \tag{8}$$

A few clarifying comments are now due with respect to the analytical integration of the trade friction and cross-country substitutability parameters into the baseline in Mihailov (2003) we summarized in this subsection.

2.2 Incorporating Iceberg Costs

Although heavily exploited in many NOEM models, the key *pricing-to-market* (PTM) assumption – which changes crucially their equilibrium outcomes – has not yet received an explicit and solid grounding within this line of literature. To rationalize *market segmentation* and the ensuing *possibility* for PTM behavior by monopolistically competitive firms, we introduce *symmetric* costs of international trade in goods, $\tau\,(\equiv \tau^*)$, in the set-up under CCP vs. PCP analyzed in Mihailov (2003). Following Obstfeld and Rogoff's (2001) NOEM application of ideas in the traditional literature,[9] we model them as being of the "iceberg" type, i.e. real losses in transit expressed in per cent of the quantity shipped: $0 \leq \tau < 1$. Although we model our τ parameter in a quite literal, "melting iceberg" fashion, we would nevertheless wish to interpret it in a much more general context, essentially capturing all kinds of frictions or impediments to international trade (or transaction costs, in a still broader sense). These may normally range from obstacles of a subjective (policy) nature such as tariff and non-tariff barriers to considerations of an objective (physical) character such as transport costs that are themselves a function of distance and transportation technology. Productivity shocks are abstracted away in our present framework[10] so the quantities of goods exchanged in equilibrium and, hence, national trade-to-output ratios, are determined by relative monetary shocks.

In both our CCP and PCP versions, the iceberg cost parameter $\tau \in [0, 1)$ enters the model via firms' production cost structure. Under this assumption a fixed fraction τ of each good shipped abroad "melts" in transit. Therefore

[9] Exogenous real *"iceberg" costs* of international trade originate in the modelling approach common to the Ricardian comparative advantage trade and payments theory: to mention just the most prominent classic studies, in Samuelson (1952) and Samuelson (1954). Transport costs of that type are assumed too in the seminal paper by Dornbusch, Fisher and Samuelson (1977) and its NOEM interpretations in Obstfeld and Rogoff (1996: Chapter 4, Section 5, pp. 235-257) and Kraay and Ventura (2002). Trading frictions, not necessarily modelled as iceberg costs, have also recently been employed outside NOEM, by Martin and Rey (2000), Sercu and Uppal (2000), Parsley and Wei (2000) and Betts and T. Kehoe (2001), among others.

[10] Within the (New-)Keynesian modelling perspective of which we make use here this is not so unusual since output is anyway demand-determined.

firms have to also produce the *additional output* that is eventually lost when crossing the "ocean", given that there is demand corresponding to the remaining part of the output produced for export. A wedge of τ is consequently driven between output *produced* and output *consumed* in *real* terms once iceberg costs are considered in a NOEM set-up of the kind.

With view to this, for a *real* (*Foreign*) import demand of $c^*_{i,s}$, a *Home* firm $i \in [0,1]$ must ensure (and hence, produce) a *real* (*Home*) export supply of $\frac{c^*_{i,s}}{1-\tau}$.[11] A simple calculation shows why: a real quantity of $\frac{c^*_{i,s}}{1-\tau}$ is produced and shipped abroad from which only $c^*_{i,s}$ arrives and is consumed by the importing consumer. The difference:

$$\frac{c^*_{i,s}}{1-\tau} - c^*_{i,s} = \tau \frac{c^*_{i,s}}{1-\tau} \tag{9}$$

"melts" in transit, so *real* losses due to such a trade friction are a constant fraction τ of the amount shipped by the exporting producer.

2.3 Distinguishing Brand from Type Substitutability

In Bacchetta-van Wincoop's (1998, 2000 a, b, 2001) context of frictionless trade the extent to which agents would substitute away from imports and into home product analogues once trade costs are accounted for would depend on a *unique* consumption substitutability parameter. Had we retained their assumption, as in Mihailov (2003), that the *same* type of product is being produced (although in different brands) in the two countries modelled, implying a *very high* elasticity of cross-country output demand, equilibrium trade would be (close to) zero because of the optimally arising home bias in goods consumption, as we formally show further down. To allow for a richer setting where households may not be as willing to substitute away from imports, in the present paper we also add to $\varphi\,(\equiv \varphi^*)$, the elasticity of substitution between any two *differentiated brands*, a distinct *composite-good type* substitutability parameter ν, $0 \leq \nu \leq \varphi > 1$. In separating product *type* from *brand* consumption substitutability our approach here is similar to that in recent NOEM contributions such as Obstfeld-Rogoff (1998), Galí-Monacelli (2002) and notably Tille (1998, a, b, 2001, 2002).

This substitutability decomposition proves to be a useful analytical device. It allows us to distinguish trade between countries producing the same, but *diversified* across brands, output type ($\nu = \varphi > 1$), as in Mihailov (2003), from trade between countries *specializing* in only one of two different output types, each diversified across *national* brands ($\nu < \varphi > 1$). In a more general sense or as a metaphor, we could refer to these alternative extremes as complete *diversification* of (world) production and complete *specialization* of (national) production. We do not otherwise deviate from the symmetry considered in Mihailov (2003).

Our model thus conveniently *nests* two conceptually different types of international trade, namely the exchange of *similar* vs. *different* "output mixes".

[11] The logic for a *Foreign* firm $i^* \in (1,2]$ is, certainly, symmetric.

To our knowledge, they have not been explicitly compared within a coherent framework in the existing literature, with Tille (2002) providing a very recent exception. As a consequence, our NOEM set-up readily reduces either to the above-cited Bacchetta-van Wincoop's series of articles featuring trade in similar products (the case of $\nu \equiv \varphi > 1$) or to the more frequent alternative focus on complete specialization and trade in different national good-types, but with *unit* substitutability between the latter (when $1 \equiv \nu < \varphi$) employed in most NOEM papers following Corsetti-Pesenti (1997, 2001 a, b, 2002). Although also retaining the restrictive assumption of unit substitutability, Tille's (2002) analysis allows for even greater generality than our present study by introducing two sectors in each of the two countries and by varying sectors' relative size. Yet he does not explore how transport costs and non-unitary substitutability in national output composites influence trade prices and flows, which we do here.

In both our CCP and PCP versions, the substitutability parameters ν and φ, with $0 < \nu < \varphi > 1$, enter the model via the symmetric *preference* structure embodied in (2). Following NOEM modelling tradition, φ is assumed to be *larger* than 1.[12] In general, we further down assume that $\nu < \varphi > 1$. Such an assumption seems the appropriate one in our stylized model context. The reason is that $\nu < \varphi$ implies that there is less substitutability across the aggregate national outputs of the two countries than between any two differentiated brands produced in each of these countries, because of naturally (geographically) predetermined complete national *specialization* in production. Consumption substitutability is thus *lower across types than across brands* in the unified international trade framework we study.[13] Moreover unlike φ, ν is not restricted to the elastic region of its domain only, a feature that is related to some lasting debates in the empirical trade and development literature[14] and that has important theoretical implications in our further analysis.

3 Costly Trade under CCP vs. PCP

In this section, we compare across our *invoicing-specific* model versions and under *float* and *symmetry* the optimization problems agents solve and the resulting equilibrium. In particular, the outcomes for the exchange-rate level, international relative prices, cross-country consumption and leisure allocations and, ultimately, some key measures of trade flows are derived and interpreted.

[12] The reason is that otherwise the marginal revenue of firms will be negative (see, for instance, Obstfeld and Rogoff (1996), Chapter 10, footnote 2, p. 661).

[13] In the special case of $\nu = \varphi$, our two elasticity parameters coincide so that the set-up reduces to world production of the same homogeneous good type that takes place in both countries and is *diversified* across brands, as in Mihailov (2003).

[14] More precisely, a number of studies have argued that world demand for many products, in particular primary commodities, is income- and price-inelastic. This has also been advanced as a major explanation behind the secular decline in the terms of trade of such goods. Todaro and Smith (2002), p. 522, for instance, refer to World Bank (1994), Table 2.5, in their popular textbook to claim that the elasticity of demand for foodstuffs to developed countries' income changes is 0.6% and of agricultural raw materials such as rubber and vegetable oils 0.5%.

The essential algebra underlying our main results is systematized in more detail in appendices *A.1* and *B*.

3.1 Optimization and Equilibrium

Consumption Demands and Price Levels Maximizing (2) subject to the constraints (3) through (6) derives the optimal demands for H- (equation (10) below) and F-produced ((11) below) goods and the respective price indices at the domestic absorption (equation (12)), import demand (13) and consumer (14) levels for the CCP vs. PCP model versions[15] as follows:

$$c_{H,s}^{C} = \frac{1}{2}\left(\frac{P_H^C}{P^C}\right)^{-\nu}\frac{M_s}{P^C} \quad \text{vs.} \quad c_{H,s}^{P} = \frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}; \tag{10}$$

$$c_{F,s}^{C} = \frac{1}{2}\left(\frac{P_F^C}{P^C}\right)^{-\nu}\frac{M_s}{P^C} \quad \text{vs.} \quad c_{F,s}^{P} = \frac{1}{2}\left(\frac{\overbrace{\frac{S_s^P P_F^{*,P}}{1-\tau}}^{\equiv P_{F,s}^P}}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}; \tag{11}$$

with

$$P_H^C \equiv \left[\int_0^1 \left(P_i^C\right)^{1-\varphi} di\right]^{\frac{1}{1-\varphi}} \quad \text{vs.} \quad P_H^P \equiv \left[\int_0^1 \left(P_i^P\right)^{1-\varphi} di\right]^{\frac{1}{1-\varphi}}; \tag{12}$$

$$P_F^C \equiv \left[\int_1^2 \left(P_{i^*}^C\right)^{1-\varphi} di^*\right]^{\frac{1}{1-\varphi}} \quad \text{vs.} \quad \underbrace{\frac{S_s^P P_F^{*,P}}{1-\tau}}_{\equiv P_{F,s}^P} \equiv \left[\int_1^2 \underbrace{\left(\frac{S_s^P P_{i^*}^{*,P}}{1-\tau}\right)}_{\equiv P_{i^*,s}^P}{}^{1-\varphi} di^*\right]^{\frac{1}{1-\varphi}}; \tag{13}$$

$$P^C \equiv \left[\frac{1}{2}\left(P_H^C\right)^{1-\nu} + \frac{1}{2}\left(P_F^C\right)^{1-\nu}\right]^{\frac{1}{1-\nu}} \quad \text{vs.} \tag{14}$$

$$P_s^P \equiv \left[\frac{1}{2}\left(P_H^P\right)^{1-\nu} + \frac{1}{2}\underbrace{\left(\frac{S_s^P P_F^{*,P}}{1-\tau}\right)}_{\equiv P_{F,s}^P}{}^{1-\nu}\right]^{\frac{1}{1-\nu}}.$$

[15] Indicated by a superscript of C or P, respectively. For more deatils on our invoicing-specific notation see Mihailov (2003).

Output Prices The expected market value of real profits[16] which a H firm $i \in [0,1]$ maximizes under CCP vs. PCP is defined by:

$$\underset{P_i^C, P_i^{*,C}}{Max} E_0 \left[\frac{u_{c,s}}{P^C} \underbrace{\left(P_i^C c_{i,s}^C + S_s^C P_i^{*,C} c_{i,s}^{*,C} - W_s^C c_{i,s}^C - \frac{W_s^C c_{i,s}^{*,C}}{1-\tau} \right)}_{\equiv \Pi_{i,s}^C} \right], s \in S \quad (15)$$

$$\text{vs. } \underset{P_i^P}{Max} E_0 \left[\frac{u_{c,s}}{P_s^P} \underbrace{\left(P_i^P c_{i,s}^P + P_i^P c_{i,s}^{*,P} - W_s^P c_{i,s}^P - \frac{W_s^P c_{i,s}^{*,P}}{1-\tau} \right)}_{\equiv \Pi_{i,s}^P} \right], s \in S. \quad (16)$$

Using the first order conditions of the two problems, the CCP vs. PCP optimal prices preset by the Home firm i, which is also the representative Home firm, for consumer households in the domestic and foreign markets are thus, respectively:

$$P_i^C = P_H^C = \frac{\varphi}{\varphi - 1} \frac{E_0 \left[u_{c,s} W_s^C M_s \right]}{E_0 \left[u_{c,s} M_s \right]} \text{ vs.} \quad (17)$$

$$P_i^P = P_H^P = \frac{\varphi}{\varphi - 1} \frac{E_0 \left[\frac{u_{c,s}}{P_s} W_s \frac{M_s}{P_s^{1-\nu}} \right] + (1-\tau)^{\nu - 1} E_0 \left[\frac{u_{c,s}}{P_s} W_s \frac{S_s M_s^*}{(S_s P_s^*)^{1-\nu}} \right]}{E_0 \left[\frac{u_{c,s}}{P_s} \frac{M_s}{P_s^{1-\nu}} \right] + (1-\tau)^{\nu - 1} E_0 \left[\frac{u_{c,s}}{P_s} \frac{S_s M_s^*}{(S_s P_s^*)^{1-\nu}} \right]}; \quad (18)$$

$$P_i^{*,C} = P_H^{*,C} = \frac{1}{1-\tau} \frac{\varphi}{\varphi - 1} \frac{E_0 \left[u_{c,s} W_s^C M_s^* \right]}{E_0 \left[u_{c,s} S_s^C M_s^* \right]} \text{ vs.} \quad (19)$$

$$P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P (1-\tau)} \quad \Rightarrow \quad \underbrace{P_s^{*,P} = \left[\frac{1 + \left(\frac{1}{1-\tau} S_s^P \right)^{1-\nu}}{1 + \left(\frac{1}{1-\tau} \frac{1}{S_s^P} \right)^{1-\nu}} \right]^{1-\nu} P_s^P.}_{\text{PPP-}related\text{ equation}} \quad (20)$$

As in Mihailov (2003), under PCP the exchange-rate pass-through to import prices is *unitary*, while under CCP it is *zero*. For the same reason, the

[16] Note that the relevant weights for the states of nature in the formulas below are related to the marginal utility of consumtion of the representative Home shareholder, $u_{c,s}$.

(Home) CPI is constant under CCP, P^C, but state-dependent under PCP, P_s^P. With transport cost and distinct cross-country substitutability incorporated in the present extended model, one should observe the following modifications in the corresponding formulas. First, equation (11) shows that, irrespective of the invoicing assumption, import demand now optimally depends on τ as well, via the prices P_F^C and $P_{F,s}^P$.[17] Note that under both CCP and PCP τ enters the *consumer* price (cif) but this latter price is preset under CCP in the currency of the destination market, while under PCP it is the price *excluding* transportation costs (fob) which is preset in the national currency of the producer. Consequently, the corresponding PCP *consumer* price becomes sensitive to the exchange rate and is, in such a way, "flexibilized". Second, optimal consumer demands (10) and (11) reveal that it is now ν that matters for cross-country substitution, although φ is still important in the determination of CPIs.[18]

The cost of international exchange, τ, is thus ultimately passed on to consumers, via the effective consumer price, but in a different way under the alternative invoicing conventions we study. Under *CCP* it is passed on to importing foreign consumer-households via the price charged directly in *foreign* currency. The exchange-rate risk is nevertheless borne by domestic producing firms, because of their preset export-market prices. Under *PCP* the trade cost is passed on to importing foreign consumer-households too, but now the mechanism is not the same. It consists in buying, at the price charged in the *seller's* currency, the equivalent – including the output to be lost in transit – of the quantity of imports optimally demanded. Then the buyer loses τ% of the shipped quantity, so that he effectively consumes less in *real* terms than the amount paid for.[19]

As evident from (20), the price at which Home representative firm's product sells in Foreign under PCP, $P_{H,s}^{*,P}$, depends on the exchange-rate level, S_s^P. But unlike the frictionless, unique substitutability case analyzed in Mihailov (2003), PPP does not hold anymore in the present PCP model version. Nevertheless, there is still an equation reminiscent of PPP, with a much more complicated function replacing the exchange rate. Note that once trade frictions are accounted for, CPIs can be equalized only under two conditions easily verified in (20): (i) $S_s^P = 1$ (peg) and/or (ii) $\nu = 1$ (unit cross-country substitutability).

Equilibrium The constrained optimization problems agents solve and the market clearing conditions for the world economy given the invoicing and timing assumptions of our stochastic NOEM framework lead to an equilibrium concept consistent with the described environment. Since it is not essentially different from the one in Mihailov (2003), its formal definition is relegated to *Appendix A.2*, while the equilibrium solutions for the macrovariables we are interested in

[17] The CCP export market price for Foreign, P_F^C, is optimally preannounced at a level symmetric to expression (19) for the analogous price for Home, $P_H^{*,C}$, as can also be verified from ($8_F^H C$) in our CCP Summary Table 4F of *Appendix A.1*.

[18] Which becomes clear from the price level formulas (12) through (14) above.

[19] An alternative interpretation could be that importing households pay a *higher* "true" price for the consumed quantity, because they also buy the quantity lost in transit and thus not consumed.

are presented and discussed in the following subsections.

3.2 Equilibrium Nominal Exchange Rate

The equilibrium nominal exchange rate (NER)[20] solves the international *forex market clearing* condition which states that excess supply of each of the two currencies (expressed in the same monetary unit) is zero in any state of nature $s \in S$. Given the *full symmetry* we assumed, i.e. with $P_H^C = P_F^{*,C}$, $P_F^C = P_H^{*,C}$, $P^C = P^{*,C}$ under CCP vs. $P_H^P = P_F^{*,P}$, $P_{F,s}^P \equiv \frac{S_s^P P_F^{*,P}}{1-\tau}$, $P_{H,s}^{*,P} \equiv \frac{P_H^P}{(1-\tau)S_s^P}$, $P_s^P = \left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}} P_s^{*,P}$ under PCP[21] it can be derived to be[22]

$$S_s^C = \frac{M_s}{M_s^*} \text{ vs. } S_s^P = \left[\underbrace{\frac{1+(1-\tau)^{1-\nu}\left(S_s^P\right)^{1-\nu}}{(1-\tau)^{1-\nu}+\left(S_s^P\right)^{1-\nu}}}_{\equiv\phi(S_s^P;\varphi,\nu,\tau)}\right]^{\frac{1}{\nu}} \left(\frac{M_s}{M_s^*}\right)^{\frac{1}{\nu}}. \tag{21}$$

The exchange rate expression (21) under CCP, S_s^C, is exactly the same as the one in Mihailov (2003), so under full symmetry neither transport cost nor distinct substitutability considerations affect CCP NER determination in equilibrium. The reason is that import prices and, hence, CPIs are preset independently from the ex-post NER at the same level in Home and in Foreign. The PCP exchange rate however, S_s^P, is now much more complicated (but again implicit) function.

With a fixed exchange-rate regime (i.e. when $M_s \equiv M_s^*$, $\forall s \in S$) the CCP NER becomes directly 1 for any possible state of nature, whereas with a peg under PCP the equilibrium NER expression (21) reduces to[23]

$$(1-\tau)^{1-\nu}\left(S_s^P\right)^{\nu} - (1-\tau)^{1-\nu}\left(S_s^P\right)^{1-\nu} + S_s^P - 1 = 0. \tag{22}$$

$S_s^P = 1$ is clearly a solution for any ν. Simulations have confirmed that equation (22) has always the same *unique* solution, $S_s^P = 1$, once ν is given some numerical value and the roots are restricted to *positive* (real) numbers, as it should be.[24] So in the present context with an iceberg friction and two distinct substitutabilities a *peg* implies again that – under CCP as well as under PCP and ex-post as well ex-ante – the exchange rate can be substituted by 1 in all expressions which contain it, a finding we shall exploit further on in discussing the effects of a fixed exchange-rate regime on trade prices and flows.

[20] Defined as the *Home*-currency price of *Foreign* money.
[21] With also $\nu < \varphi = \varphi^* > 1$ and $0 < \tau = \tau^* < 1$ in both model versions.
[22] See *Appendix A.3*.
[23] See again *Appendix A.3*.
[24] Since the exchange rate is defined only for positive values, $S_s > 0$.

Optimal Firm Prices under Full Symmetry Using (7) and its equivalent for Foreign as well as (21) to substitute for the endogenous variables W_s, W_s^* and S_s in (17) through (20), the optimal firm prices derived earlier can now be fully determined under CCP and (via the implicit function giving the equilibrium NER) PCP. The final model solutions for prices in terms of exogenous variables and parameters only are thus:

$$P_i^C = P_H^C = \frac{\varphi}{\varphi - 1} P^C \frac{E_0\left[u_{l,s} M_s\right]}{E_0\left[u_{c,s} M_s\right]} \text{ vs.}$$

$$P_i^P = P_H^P = \frac{\varphi}{\varphi - 1} \frac{E_0\left[u_{l,s} \frac{M_s}{P_s^{1-\nu}}\right] + (1-\tau)^{\nu-1} E_0\left[u_{l,s} \frac{S_s M_s^*}{\left(S_s P_s^*\right)^{1-\nu}}\right]}{E_0\left[\frac{u_{c,s}}{P_s} \frac{M_s}{P_s^{1-\nu}}\right] + (1-\tau)^{\nu-1} E_0\left[\frac{u_{c,s}}{P_s} \frac{S_s M_s^*}{\left(S_s P_s^*\right)^{1-\nu}}\right]};$$

$$P_i^{*,C} = P_H^{*,C} = \frac{1}{1-\tau} \frac{\varphi}{\varphi - 1} P^{*,C} \frac{E_0\left[u_{l,s} M_s^*\right]}{E_0\left[u_{c,s} M_s\right]} \text{ vs.}$$

$$P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P (1-\tau)} \quad \Rightarrow \quad \underbrace{P_s^{*,P} = \left[\frac{1 + \left(\frac{1}{1-\tau} S_s^P\right)^{1-\nu}}{1 + \left(\frac{1}{1-\tau} \frac{1}{S_s^P}\right)^{1-\nu}}\right]^{1-\nu} P_s^P}_{\text{PPP-}\textit{related}\text{ equation}}.$$

It is easily seen that under *CCP* the prices set by the Home representative firm domestically, P_H^C, and abroad, $P_H^{*,C}$, will generally not be the same with now nonzero iceberg costs $0 < \tau < 1$ even if $E_0\left[u_{l,s} M_s\right] = E_0\left[u_{l,s} M_s^*\right]$ is true, as it is under *separable* utility in consumption and leisure.[25] It is also clear that under *PCP* and float when just one price, in the domestic currency, is optimally prefixed in each country, the two preannounced prices in the model will have the same level, $P_H^P = P_F^{*,P}$ (given symmetry and separability, again). Yet the respective ex-post PCP prices in the foreign currency, $P_{H,s}^{*,P}$ and $P_{F,s}^P$, will in general not be equal to those preset domestically. Observe as well that in the presence of iceberg costs ($0 < \tau < 1$), a *peg* will *never* guarantee that the relevant (ex-post) prices of home and foreign output agents in both countries face under CCP as well as under PCP are the same, i.e. that $P_H^C = P_H^{*,C} = P_F^C = P_F^{*,C}$ and $P_H^P = P_H^{*,P} = P_F^P = P_F^{*,P}$. This is a result very different from – in a sense, opposite to – what one would obtain in the frictionless model version considered in Mihailov (2003).

We are now ready to derive – under full symmetry and separability – expressions for some traditional characteristics of international trade which we interpret below.

[25] This separability condition was formally proved by Bacchetta and van Wincoop (2000 a): see their Lemma 1 and related Proposition 1.

3.3 Equilibrium Relative Prices

Let us begin by comparing across our invoicing conventions the three most important pairs of international relative prices. This analysis will help us later in understanding the channel along which optimal consumption – and, hence, trade – flows are determined in our NOEM set-up.

Relative Price of Foreign to Domestic Goods Under CCP with jointly symmetric money shocks and separable preferences, the relative price of foreign-produced goods in terms of domestically-produced ones in both countries is *preannounced* at the *same* level of $\frac{1}{1-\tau}$. In such a way, any effects of the ex-post values of these key international relative prices on consumer behavior are *precluded* under CCP:[26]

$$p_H^C \equiv \frac{P_F^C}{P_H^C} = \frac{1}{1-\tau} = \frac{P_H^{*,C}}{P_F^{*,C}} \equiv p_F^{*,C} \text{ for } \forall s \in S. \tag{23}$$

Under PCP, the resulting relative prices of foreign-produced goods in terms of domestically-produced ones are generally *not reciprocal* across countries any-more, as it was in our frictionless baseline, just because of the *nonzero* iceberg costs ($\tau \neq 0$):

$$p_{H,s}^P \equiv \frac{\overbrace{\frac{S_s^P P_F^{*,P}}{1-\tau}}^{\equiv P_{F,s}^P}}{P_H^P} = \frac{S_s^P}{1-\tau} \neq \frac{1}{S_s^P\,(1-\tau)} = \frac{\overbrace{\frac{P_H^P}{S_s^P\,(1-\tau)}}^{\equiv P_{H,s}^{*,P}}}{P_F^{*,P}} \equiv p_{F,s}^{*,P} \text{ unless } S_s^P = 1. \tag{24}$$

Terms of Trade With $0 < \tau < 1$, the terms of trade (ToT) are still *inversely* defined with respect to our symmetric *countries* under CCP, like it was in the baseline model without trade frictions, but not anymore under PCP. However, the *inverse* relationship in the ToT definitions across *price setting* in terms of the exchange rate remains valid in the present extended set-up too, due to the invoicing-specific import and export price indexes. Our *CCP* model version thus implies, even with iceberg costs, a *negative* relationship between the nominal exchange rate (NER) and the ToT: a nominal depreciation *improves* the terms of trade. By contrast, in our *PCP* model version the relationship between the NER and the ToT is *positive*, so that a nominal depreciation *weakens* the terms of trade and induces *expenditure switching*, similarly to the vast literature in the Mundell-Fleming-Dornbusch tradition:

[26] This is a result *analogous* to the corresponding one in the frictionless baseline, the only difference being that then we logically had $\frac{1}{1-\tau} = \frac{1}{1-0} = 1$.

$$(ToT)^C_{H,s} \equiv \frac{\overbrace{P^C_F}^{\equiv P^{Im,C}_H}}{\underbrace{S^C_s P^{*,C}_H}_{P^{Ex,C}_H}} = \frac{1}{S^C_s} = \left(\frac{\overbrace{P^{*,C}_H}^{\equiv P^{Im,C}_F}}{\underbrace{\frac{P^C_F}{S^C_s}}_{\equiv P^{Ex,C}_{F,s}}} \right)^{-1} \equiv \left[(ToT)^{*,C}_{F,s} \right]^{-1} \neq 1 \text{ unless } S^P_s = 1 \tag{25}$$

$$\text{vs. } (ToT)^P_{H,s} \equiv \frac{\overbrace{P^P_{F,s}}^{\equiv P^{Im,P}_{H,s}}}{\underbrace{P^P_H}_{\equiv P^{Ex,P}_{H,s}}} = \frac{\frac{S^P_s P^{*,P}_F}{1-\tau}}{P^P_H} = \frac{S^P_s}{1-\tau} \neq \frac{1}{S^P_s(1-\tau)} =$$

$$= \frac{\overbrace{\frac{P^P_H}{S^P_s(1-\tau)}}^{\equiv P^{Im,P}_{F,s}}}{\underbrace{P^*_F}_{\equiv P^{Ex,P}_{F,s}}} \equiv (ToT)^{*,P}_{F,s} \text{ unless } S^P_s = 1. \tag{26}$$

The result in (26) is *new*. It implies that once transport costs are considered in a model assuming PCP, state-dependent NER deviations from the initial symmetric equilibrium of $S_0 = 1$ are *magnified* in the terms of trade a country faces. Due to the invoicing-specific import and export price index definitions again, the PCP ToT should be *more volatile* (across states of nature) than the underlying PCP NER, in equilibrium, once a symmetric trade friction is considered in a stochastic NOEM context like the one we analyze. By contrast, with $\tau = 0$ under PCP or even with $\tau \neq 0$ under CCP, the volatility of the ToT is exactly the same as that of the NER, which is evident from (25) and (26).

Real Exchange Rate With $0 < \tau < 1$, PPP now fails, as we noted earlier. Yet both our PCP and CCP model versions derive a *real* exchange rate (*R*ER) that is *inversely* defined across *countries*, just like it was in the frictionless baseline:[27]

$$(RER)^P_{H,s} \equiv \frac{S^P_s P^{*,P}_s}{P^P_s} = \underbrace{\frac{(1-\tau)S^P_s+1}{(1-\tau)+S^P_s}}_{\equiv[\phi(S^P_s;\varphi,\nu,\tau)]^{-1}} =$$

$$= \left(\frac{\frac{P^P_s}{S^P_s}}{P^{*,P}_s} \right)^{-1} \equiv \left[(RER)^{*,P}_{F,s} \right]^{-1} \neq 1 \text{ unless } S^P_s = 1 \text{ vs.} \tag{27}$$

[27] Note that with $\tau = 0$, the expression under *PCP* in the extended model here becomes state-invariant and equal to 1, and thus *identical* to the analogous expression in our frictionless baseline.

$$(RER)_{H,s}^{C} \equiv \frac{S_s^C P^{*,C}}{P^C} = S_s^C = \left(\frac{\frac{P^C}{S_s^C}}{P^{*,C}} \right)^{-1} \equiv \left[(RER)_{F,s}^{*,C} \right]^{-1} \neq 1 \text{ unless } S_s^P = 1. \tag{28}$$

3.4 Equilibrium Consumption and Leisure across Countries

Having looked at CCP vs. PCP international relative prices in equilibrium, we now turn to the corresponding cross-country real allocations. Our main results are summarized in the *propositions* we state next, in their logical order. Proofs, based largely on earlier definitions and derivations, are relegated to *Appendix B* while interpretations are suggested further down in the main text.

Proposition 1 *(Relative Consumption)* Relative *real consumption is determined by the* relative *money stock; but under* PCP *and not CCP,* trade costs *and* import demand elasticity *influence as well the equilibrium allocation across countries of the quantities consumed.*

An important modification in the conclusions with respect to our initial set-up in Mihailov (2003) is that it is now a *richer* parameter set, (τ, ν) compared to only φ earlier, which pins down relative consumption under PCP in any state of nature $s \in S$. In particular, the relevant elasticity of consumption demand is the *cross-country* one, ν with $0 \leq \nu < \varphi > 1$, and not the substitutability across the homogeneous product brands, $\varphi > 1$, as in our initial study. What is novel here, as also mentioned earlier, is that ν is defined over a larger domain, including in addition the region of import demand *in*elasticity as well as the case of *unit* elasticity, , $0 \leq \nu \leq 1 < \varphi$. This finding has insightful consequences for our analysis to which we shall return in more detail later.

Proposition 2 *(Home Bias) With* positive and symmetric trade costs *and given that $\nu \neq 0$, the optimal split-up of real consumption between demand for domestic and foreign goods always results under* CCP *in a* symmetric home bias in both countries, $(1-\tau)^{-\nu} > 1$, invariant across states *of nature; under* PCP*, by contrast, this optimal split-up is determined by the* equilibrium nominal exchange rate.

Now with iceberg costs, *import substitution* and *expenditure switching* are generally *optimal* not only under PCP when there is exchange rate pass-through but also under CCP when there isn't. Consequently, under costly trade in similar output mixes and even full symmetry, there will always be a *home* bias (reciprocal for the two countries), unless (i) $\tau = 0$ or (ii) $\nu = 0$. This home bias under elastic import demand is derived to be $(1-\tau)^{-\varphi}$, i.e. a positive function of τ and $\varphi(\equiv \nu) > 1$, the *unique* consumption substitutability across brands of the *homogeneous* good-type modelled. Due to the trading friction, $\tau \neq 0$

(with $\varphi > 1$ by definition), and *ceteris paribus*, foreign-produced goods become more expensive, hence less demanded, than their nationally-produced (close) substitutes. These conclusions are also valid under costly trade in different composite outputs (when $\nu < \varphi > 1$) with CCP but not with PCP. In that latter case, the consumption bias is not necessarily also a home bias for both countries; yet a home bias in each of the economies is likewise always generated for the realistic region of *moderate* trade costs and import demand which is (even marginally) *elastic* by *small* monetary shocks, as appropriate in studying a sticky-price environment.

Evidence for a home bias in goods consumption has often been found in applied work, and is thus empirically relevant. The theoretical reasons proposed to explain it have usually been associated with either transaction costs or structural or informational asymmetries. The NOEM literature has only recently started to integrate such a feature into its mainstream set-up. Warnock (1999), for example, imposes it via *heterogeneous preferences* of households. In our analytical framework here the home bias originates in the optimal behavior of economic agents when facing a trade friction, as Obstfeld and Rogoff (2001) first did within NOEM (under unit cross-country substitutability). The realistic home bias *rationalized* by incorporating iceberg costs into an otherwise fully symmetric two-country economy *and nuanced* across trade/output compositions and price-setting conventions is another novel feature within NOEM modelling, to which we have contributed with this paper.

Proposition 3 *(Relative Leisure) Under CCP as well as under PCP, output, employment and leisure are* not generally equal *across nations.*

The intuition behind Proposition 3 is the following. Since output is demand-determined, up to *exhausting* the CiA constraint, and technologies are assumed *identical*, the two countries do *not* generally produce the same quantities and do *not* employ the same labor in equilibrium. Hence, the hours of *leisure* the representative households enjoy in Home and in Foreign are in general *not* the same either.

3.5 Equilibrium Trade Flows

In this subsection, we interpret the *equilibrium trade flows* derived under our alternative invoicing assumptions with shipment losses in *Appendix A.4*.

Trade Shares by Country It is shown in the mentioned appendix that the *iceberg-cost augmented* trade share curve for *Home* under CCP vs. PCP is given by

$$(ft)_H^C = \frac{2}{(1-\tau)^{1-\nu} + 1} = const \lesseqgtr 1 \text{ for } \nu \gtreqless 1 \text{ vs.} \tag{29}$$

$$(ft)^P_{H,s} = \frac{2}{(1-\tau)^{1-\nu}\underbrace{\left[\left(\frac{M_s^*}{M_s}\right)^{\frac{1}{1-2\nu}}\left(\frac{P_s^P}{P_s^{*,P}}\right)^{\frac{1-\nu}{1-2\nu}}\right]}_{=S_s^P}^{\nu-1}+1} \neq const \text{ unless } S_s^P = 1. \tag{30}$$

The corresponding curves for *Foreign* are, of course, symmetric:

$$(ft)^C_{F,s} = \frac{2}{(1-\tau)^{1-\nu}+1} = const \lesseqgtr 1 \text{ for } \nu \gtreqless 1 \text{ vs.} \tag{31}$$

$$(ft)^P_{F,s} = \frac{2}{(1-\tau)^{1-\nu}\underbrace{\left[\left(\frac{M_s^*}{M_s}\right)^{\frac{1}{1-2\nu}}\left(\frac{P_s^P}{P_s^{*,P}}\right)^{\frac{1-\nu}{1-2\nu}}\right]}_{=S_s^P}^{1-\nu}+1} \neq const \text{ unless } S_s^P = 1. \tag{32}$$

The above equations compare directly the impact of our alternative price-setting assumptions on the ratio of nominal trade to nominal output. Under CCP, (29) and (31) show that the equilibrium trade share is a state-*invariant* function of the consumption bias, $(1-\tau)^{1-\nu}$, and its deeper "fundamentals", τ and ν. Moreover, trade in terms of output is the *same* for the two countries in any state of nature that has materialized. However, with positive iceberg costs it is not 1 anymore, as in the frictionless baseline in Mihailov (2003), unless cross-country output substitutability is *unitary.* For *elastic* import demand, $\nu > 1$, trade-to-output ratios are increasing in the *own* NER and smaller than 1 in both economies, due to the equal preference for domestic and foreign brands under high substitutability and trade. If import demand is instead *inelastic*, $\nu < 1$, trade shares are decreasing in the *own* NER and are both larger than 1, because of the same equal taste for both goods which are now, under national specialization of production, practically not substitutable. Under PCP, by contrast, trade-to-output ratios by country are *not* state-invariant unless relative money-stock equilibrium has occurred, as clear from (30) and (32). Trade-to-output can be 1 in both economies only with *unitary* import substitutability, just like under CCP.

To illustrate the intriguing reversal arising in the *inelastic* region, $0 < \nu < 1$, of the *PCP trade share curves* we have just highlighted, let us compare figures 1 and 2. In both figures, the curves for Home are defined by equation (30), and those for Foreign by (32) and are drawn for a baseline computation taking $\tau = 0.2$, as being close to a realistic average across real-world economic branches. Both graphs show national trade shares in output $(ft)^P_s$ (on the vertical axis) under PCP, float and full symmetry as a function of the nominal exchange rate S_s^P (on the horizontal axis).[28] The key difference is that in Figure 1 *bilateral*

[28] And ultimately of the relative money stock $\frac{M_s}{M_s^*}$ for a given value of the substitutability parameter, as in the frictionless baseline we analyzed in Mihailov (2003). Although being

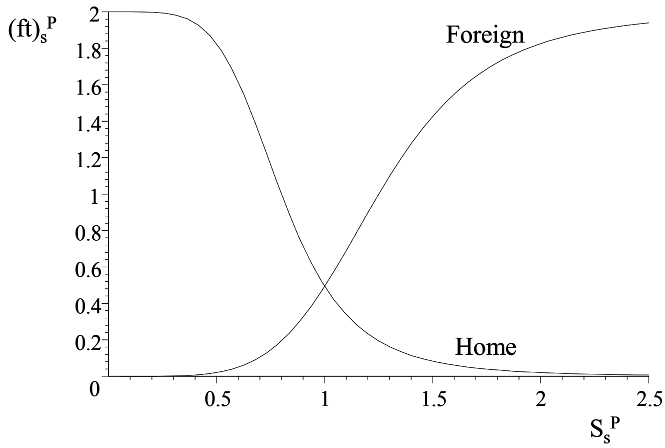

Figure 1: PCP Equilibrium Trade Share Curves under Moderate Transport Costs and Elastic Import Demand ($\tau = 0.2$ and $\nu = 6$)

import demand is assumed *elastic*, by setting $\nu = 6$, while in Figure 2 it is *inelastic*, with $\nu = 0.5$.

The *Home* trade-to-output curve is *decreasing in the equilibrium NER* for *elastic* cross-country demand (Figure 1) but *increasing* for *inelastic* cross-country demand (Figure 2). In addition, it is all over *concave* under import demand inelasticity (Figure 2) whereas if demand is elastic an inverse *logistic* curve – with only a small initial region concave – obtains (Figure 1). The *Foreign* trade share curve as a function of the *same* NER definition, S_s^P, is simply the mirror image of the Home curve, given the full symmetry in our NOEM set-up. Furthermore, if expressed as a function of the *inverse* NER, $\frac{1}{S_s^P}$, which is in fact the appropriate *own* NER definition from the viewpoint of Foreign residents, the Foreign trade-to-output curve would completely coincide with that for Home in both figures, due to symmetry again.

World Trade Share As in the frictionless case, both curves complement one another but their (equally weighted) sum does *not* add up to 1 anymore in the present extended model. Instead, (equally weighted) trade-to-output ratios sum to *less* than 1 under *diversified* production, even under peg when $S_s^P = \frac{1}{S_s^P} = 1$ for any $s \in S$ (see Figure 1). The reason is that with high cross-country substitutability a lot of imported quantities are substituted away into domestic analogues once (even symmetric) transport costs are taken into

again a function of exogenous money growth rates, the equilibrium NER is now defined by a more complicated implicit function, (21), and cannot be directly substituted away into the PCP trade share formulas above.

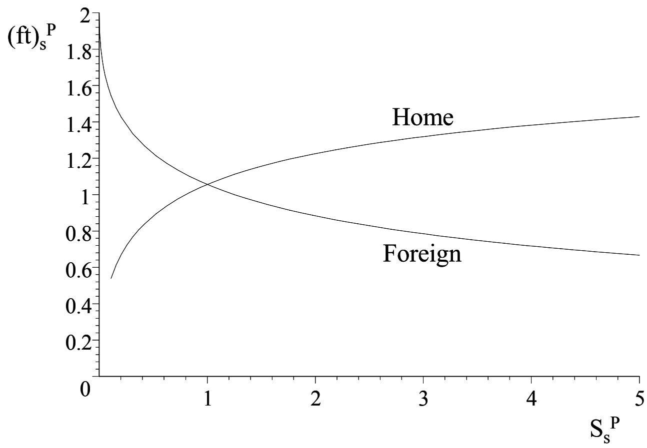

Figure 2: PCP Equilibrium Trade Share Curves under Moderate Transport Costs and Inelastic Import Demand ($\tau = 0.2$ and $\nu = 0.5$)

account, as in our study here. By contrast, substitution is practically much less possible – if not, at the extreme, almost impossible – under *specialized* production. Given the *same* preference for consuming the Home and the Foreign good in *equal quantities* in both countries, the (cif) value of trade is therefore artificially inflated so that its share in the value of GDP becomes exaggerated. As the *cost* of transporting the imported quantities is paid for but lost in transit and thus not consumed, (equally weighted) trade-to-output ratios now add up to (slightly) *more* than 1, even under peg for any $s \in S$ (see Figure 2).

4 Does the Exchange-Rate Regime Matter for Trade?

Making further use of the CCP vs. PCP equilibrium solutions under a symmetric iceberg friction and two distinct substitutabilities affecting consumption demand we characterized thus far, the present section focuses on the implications of the alternative *monetary* arrangements we study for international trade prices and flows. We analyze both the *expected* level of trade-to-output ratios (by country and for the world economy as a whole), the relevant measure of trade under *uncertainty*, and their *variability* across states of nature.

4.1 Trade-to-Output under CCP (with Float or Peg)

We derived national trade shares in equilibrium under *CCP* with *float* to be independent of the nominal exchange rate and, ultimately, of relative money

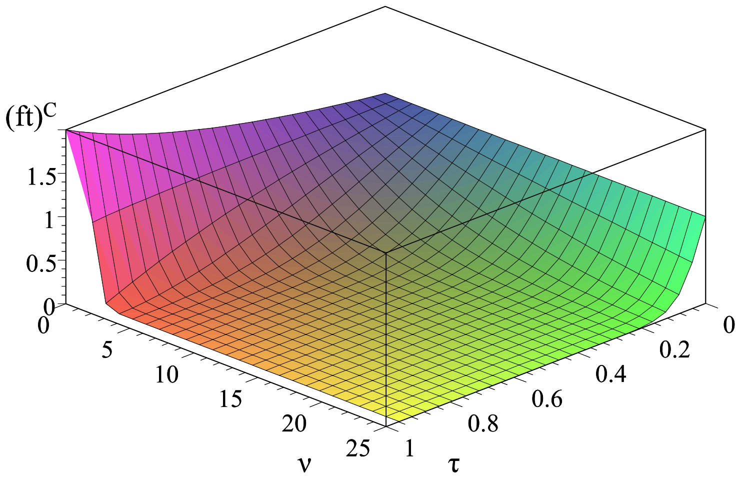

Figure 3: Peg Trade Share Surface across Iceberg Costs and Substitutabilities

stocks. CCP trade-to-output ratios are thus invariant across states of nature and coincide with their expected level. A *peg* under (full) *CCP* will therefore not change anything directly related to trade shares or their volatility in both an ex-post and ex-ante sense.

Mind, however, that under float CCP by itself does not generally imply equal equilibrium consumption, hence leisure and utility across countries. This will be the case only in the much less probable states of monetary shocks of the same magnitude. A peg under (full) CCP, by equalizing ex-post cross-country utility, will bring about this additional effect in all states of the world.

4.2 Trade-to-Output under PCP with Float

By contrast, a *peg* under *PCP*, implying $S_s^P = 1$ for any $s \in S$, will equalize the *ex-post* Home and Foreign trade shares, thus leading to a result that is essentially the same – concerning trade only, not consumption and leisure – as the one implied by (full) CCP under float. These "CCP or peg" findings on the ex-post trade share are illustrated in Figure 3. It summarizes across trade costs and substitutabilities the *identical* under CCP (with either float or peg) or peg (with either CCP or PCP) Home and Foreign *equilibrium* trade-to-output ratios.

The CCP or peg trade share function $ft^C(\tau, \nu)$ in Figure 3 is interpreted in more detail along the dimensions of its two respective determinants in section 5. But before doing it, it would be insightful to first discuss the effects of relative monetary disequilibria, possible under float, on both the *expected* level of *PCP* trade shares and their *variability* across states, which we do next.

When Does a Peg Increase Trade-to-Output? Our principal theoretical result in this paper is summarized in the following proposition.

Proposition 4 *(Expected Trade Shares under* PCP*) In addition to stabilizing trade-to-output across states of nature under* PCP*, a peg would at the same time* reduce *its expected level for both countries and, hence, for the world as a whole under* elastic *import demand but* increase *it under* inelastic *import demand.*

Given claims in the preceding literature, mostly empirical and notably exemplified by Rose (1999), as mentioned in the beginning, one would rather expect that a peg will increase trade. Theoretical work in Bacchetta and van Wincoop (1998, 2000) has warned that this is not necessarily the case. Furthermore, Mihailov (2003) has shown in a PCP extension to Bacchetta-van Wincoop's (2000) CCP benchmark that alternative price-setting assumptions impose a distinction between the effects on the exchange-rate regime on *expected* trade, which is nil under frictionless, unique substitutability symmetry, and on its *volatility*, which is important in that a peg would stabilize national trade-to-output that are state-dependent under PCP. It turns out from our present richer extension of the quoted theoretical papers that the effect of the exchange-rate regime on both expected trade shares and their variability ultimately depends on whether import demand is elastic or inelastic once some key non-monetary determinants of trade have been explicitly modelled as well, like we did here.

In interpreting our major finding, i.e. Proposition 4, we would first note that our trade measure is a *ratio*, not the value or volume of trade in general, as in many studies usually claiming that a peg would increase trade. Second, we would recall that in a *frictionless*, *unique-substitutability* setting with trade of highly similar brands of a homogeneous good-type imposing *elastic* demands for imports, as in our baseline in Mihailov (2003), the expected trade share is the same with float and peg, under CCP as well as under PCP. The introduction of more realistic features such as, in particular, costs of trade and a distinct substitutability between good-types lower than that among brands (within each type) that is, furthermore, not restricted outside the inelastic zone has thus helped enrich and clarify our understanding on the effect of the exchange-rate regime on trade measured in terms of output. Moreover, the presence of any one of these two real trade fundamentals in the extended model alone is not sufficient to produce the reversal effect of interest here, as we discuss below.

The intuition for the result in Proposition 4 we would provide is the following. We showed that a peg under PCP eliminates pass-through and, hence, expenditure switching. But in all "bad" states of nature, i.e. when a relative monetary contraction hits a country, appreciating its national currency, import substitution of cheaper imports becomes optimal for consumers under PCP;[29]

[29] We also discussed why under CCP the expenditure switching channel is closed, so no import substitution occurs at all.

the more so when substitutability is high, that is, when the brands produced by the different countries are similar. So if cross-country consumption demand is elastic, $\nu > 1$, there is some amount of import-substituting trade which is ultimately prevented by a peg, relative to a float (under the same jointly symmetric distribution of money shocks). If however, demand is inelastic, $0 < \nu < 1$, there are no incentives for consumers to substitute the domestic product by importing it in the "bad" states of nature of appreciated currency, so there is some loss of potential trade in these states under a float which a peg could somewhat "restore". We would point out to the fact that the reversal of the effect of the exchange-rate regime on expected trade-to-GDP in the inelastic region of the cross-country substitutability parameter is only possible given that trade frictions, in our case modelled through iceberg costs, are also accounted for explicitly: it is precisely the *interaction* of ν and τ that drives the result.[30] So it is the combination of the wedge driven between the cost of the domestic vs. the foreign product, intervening in decisions on import substitution, and the particular magnitude of the substitutability between these products, embodying the wish to trade and inducing, in consequence, a certain level of the equilibrium PCP NER in each state, that ultimately matters in explaining when a peg would increase trade and when a float would do it instead. Without the richer setting of the present extended model this channel of interaction could not have been uncovered, and in this consists the principal import of our theoretical work.

How Much Does the Exchange-Rate Regime Matter for Trade-to-Output? To further judge about the likely magnitude of the effect of the exchange-rate regime on expected trade we established in Proposition 4 and at the same time to explore how money-stock volatility translates into variability of the resulting trade-to-output ratios, we next simulated our model under jointly symmetric national money-growth disturbances. Having in mind that our framework was set-up assuming *price stickiness*, in line with the NOEM approach we follow here, we were interested in, and imposed in the simulation, *low* monetary uncertainty. The outcomes across a few sets of parameter constellations are reported in Table 1.[31]

Now looking at the last column of Table 1, the first regularity one notices is related to the sign of what we have defined as the gain for expected world trade

[30] As clear from the changing sign of the second term of $F''(1)$ in the Proof of Proposition 4 in Appendix B, the first one being always positive.

[31] The money-stock growth shocks (μ_s, μ_s^*) underlying the numbers in Table 1 were simulated 100 times from two independent (continuous) uniform distributions on the unit interval, $\mathcal{U}(0,1)$, one for Home and the other for Foreign. We then *centered* the shocks around 0, according to what we assumed in our no-(productivity)-growth NOEM model here, and *discretized* them. In discretizing, we used a *small* step to obtain "realistic" uncertainty, i.e. with a great (in fact, $10201 = 101 \times 101$) number of possible states, but at the same time limited the range of the latter to comply with price rigidity, namely by using $\mathcal{U}_l(-5.0, -4.9, ..., -0.1, 0, 0.1, ..., 4.9, 5.0)$. The equally-spaced values inside the parentheses defining the uniform distribution, the same for Home and Foreign, we simulated are directly interpretable as *growth rates* of the money stock in *percentages*, that is, as -0.1% or 4.9%, for instance. The GAUSS programs as well as more details on the algorithm of computations and on the results are available upon request.

$(\mu_s, \mu_s^*) \in \mathcal{U}_l(-5,5)$	$(M_s, M_s^*)\,; \nu, \tau$ - *determined* NER: *PCP-cum-Float* Mean	SD	*Cif* Trade Shares in Output, % *PCP-cum-Float* Mean H	F	SD H	F	*CCP*$\Leftrightarrow$*Peg constant* $H = F$	*Peg Gain* for *World* Trade over Float, %
PANEL I: (very) *low* transport costs: $\tau = 0.01$								
$\nu = 11$	0.9997	0.0035	95.14	94.82	1.74	1.74	94.98	−0.0016
$\nu = 2$	0.9983	0.0200	99.59	99.40	1.00	1.00	99.50	−0.0001
$\nu = 0.5$	0.9957	0.0800	100.06	100.44	2.00	2.00	100.25	0.0001
$\nu = 0.2$	1.0008	0.2005	99.66	101.14	7.84	7.84	100.40	0.0025
PANEL II: *moderate* transport costs: $\tau = 0.2$								
$\nu = 11$	0.9998	0.0021	19.43	19.36	0.37	0.37	19.39	−0.0161
$\nu = 2$	0.9984	0.0190	88.98	88.80	0.94	0.94	88.89	−0.0011
$\nu = 0.5$	0.9957	0.0760	105.39	105.75	1.89	1.89	105.57	0.0019
$\nu = 0.2$	0.9970	0.1491	108.32	109.42	5.85	5.85	108.90	0.0281
PANEL III: (very) *high* transport costs: $\tau = 0.6$								
$\nu = 11$	0.9998	0.0019	0.02	0.02	0.00	0.00	0.02	−0.0223
$\nu = 2$	0.9986	0.0165	57.21	57.08	0.67	0.67	57.14	−0.0042
$\nu = 0.5$	0.9960	0.0654	122.36	122.65	1.55	1.55	122.51	0.0047
$\nu = 0.2$	0.9956	0.0835	134.79	135.33	2.92	2.92	135.09	0.0252

Table 1: Gains from Peg/Float for World Trade: Simulation Summary

as a share in world output from a peg regime relative to a float. This measure is simply the percentage difference between the peg trade share constant and the expected trade-to-GDP under float, the latter taken as a base (i.e. normalized to 100). A positive difference is thus a trade gain from fixed exchange-rate regime whereas a negative sign means the opposite, namely that a flexible exchange-rate regime would bring about more international trade relative to world output. The simulation has thus, first of all, cross-checked and confirmed our conclusions that were formally proved in Proposition 4: under *elastic* demand (i.e. for $\nu = 11$ and $\nu = 2$ in Table 1, no matter what the particular value of τ is) a peg does reduce trade, but *only slightly* – and this is the new point here, coming out from the simulation; and under *inelastic* demand, it does increase expected trade share in GDP for the world economy as a whole, but – again – *only slightly.* "Only slightly" means more precisely by less than 1%, i.e. up to about 3 basis points,[32] as clear from the table.

Our qualitatively important theoretical finding has thus "crashed" into a quantitatively insignificant magnitude: for all practical or policy purposes, it will therefore be difficult to rely on the above result... Yet this particular quantification of the *first*-order effect, on *expected* trade-to-output, of a *second*-order model feature, namely monetary *uncertainty* embodied in the driving shocks, does not go astray from similar conclusions in related NOEM papers. It is true that the magnitude of the exchange-rate effect is tiny compared to the impact of

[32] A basis point is $\frac{1}{100}$ of 1%.

trade costs or import demand elasticity themselves. But that is a point common for the whole literature about the effects of uncertainty: on trade shares, in our case, but also on the conduct of monetary policy or on the welfare of peg. vs. float. Once uncertainty is driven by *monetary* shocks – as in our present work but also in research by others, for instance Devereux and Engel (1998, 1999, 2000) – these shocks are just not that large empirically (or when simulated, as we did here, to comply with our assumption of sticky prices), hence there is no way that they will result in a large effect.

The key import of our extended analysis is thus in the conclusion that under PCP with trade costs interacting with somewhat more structured preferences (and, ultimately, import demand), monetary uncertainty has a first-order effect on trade-to-GDP, be it a tiny one, whereas in a frictionless or single-substitutability model – e.g. Mihailov (2003) or Bacchetta - van Wincoop (2000) – such a channel cannot be captured and explained. Moreover, our quantification of the effect of peg vs. float on the expected trade share is completely of the order of magnitude of similar effects, such as those recently reported in a policy-oriented paper by Devereux, Engel and Tille (2003), for example. To quote them exactly, on page 237 of the cited reference they write: "The actual gains in expected consumption and reduction in expected employment are small (e.g. when $\rho = 2$, the gain in expected consumption in Europe is 0.02%, and the reduction in expected employment in Europe and the United States is 0.004%)".

5 The Role of Trade Costs and Import Demand Elasticity

We finally turn to the role of major *real* determinants such as trade costs and import demand (in)elasticity in the determination of trade shares, as essentially reflected in Figure 3 and Table 1.

5.1 Trade Frictions

The magnitude of the trade friction in combination with that of import demand elasticity defines the consumption bias $(1-\tau)^{1-\nu}$ and, ultimately, the state-*invariant* trade share $(ft)^C$ under *CCP* or *peg.* Our Proposition 4 has shown that under float the expected trade share is slightly higher or lower this *constant*, depending on whether import demand is inelastic or elastic, respectively.

We first examine the impact of transport costs. Figure 4, in fact a two-dimensional variation of Figure 3, shows the peg trade share (in Home as well as in Foreign) as a function of τ for seven different levels of ν. We can see that for given *elastic* import demand, *higher* transport costs *decrease – decreasingly* for $\nu = 11$ or $\nu = 6$ and *increasingly* for $\nu = 2$ or $\nu = 1.25$ – the *expected* level of

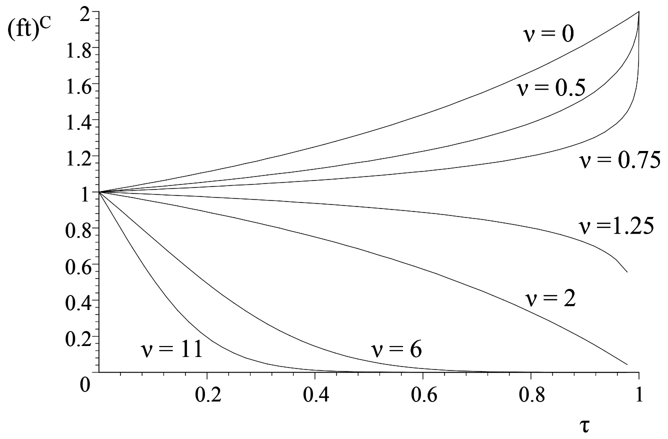

Figure 4: Peg Trade Share Curves across Iceberg Costs for Given Substitutability

trade-to-output.[33] But *inelastic* imports ($\nu = 0.75$ or $\nu = 0.5$ or $\nu = 0$) reverse this conclusion: as evident on the upper part of figure 4, higher trade frictions always lead to an increasing – but "inflated", as we are going to next make it clear – growth (see also the last but one column in all three panels of Table 1).

The above reversal result is not so intuitively natural and needs a word of comment. In such a parameter region national output mixes are so *poor substitutes* that both countries are "doomed", by their taste (or need) for diversity under complete specialization, to trade even when shipping losses are very high (or exogenously rise). For a given level of transport costs under float, the trade share in output would thus be almost insensitive to ex-ante price discrimination under CCP or ex-post expenditure switching under PCP, since households practically cannot substitute away from imports into the (now completely different) home-produced good-type. The essential reason for our finding that, even under peg, with inelastic import demand national trade shares would both be higher than 1 (see again Figure 2) is that the *value* of *nominal* trade taking account of shipment costs (*cif*) divided by *nominal* output was used to derive them.[34] This latter ratio is highly inflated by the "true" price to the consumer of the huge percentage of output lost in transit.[35]

[33] It follows from our analysis that *space* (or geography) matters, as in *gravity* models of trade, in particular if transport costs are modelled to be some positive function of distance (as we have implicitly assumed here).

[34] Recall (34) under CCP and (35) under PCP in *Appendix A.4* for Home; the respective definitions for Foreign are, of course, symmetric.

[35] Note that there is no way to measure instead trade-to-GDP in terms of the *exchanged quantities ultimately consumed relative to the produced ones* in each country, $\frac{c^*_{H,s}+c_{F,s}}{c_{H,s}+\frac{c^*_{H,s}}{1-\tau}}$ for

For the moderate ($\tau = 0.2$) to high ($\tau = 0.6$) iceberg costs we are particularly interested in, simulations have furthermore indicated that lower shipment losses tend to increase trade share *volatility* (which becomes clear from comparing the standard deviation columns of panels II and III in Table 1).[36]

As already discussed, another major role of transport costs is to generate a home bias in goods consumption. Even when monetary policies are coordinated under a peg, or in a monetary union, and the participating countries fully symmetric, there is in general an *optimally* lower demand for foreign-produced products. Such a home bias originates in the trade friction and thus independently of the specification of preferences (Proposition 2). A higher cross-country consumption substitutability then additionally exacerbates it (Proposition 2, again).

5.2 Cross-Country Substitutability

We now summarize how the degree of substitutability affects trade. Figure 5 – which is another two-dimensional perspective of Figure 3 earlier – shows that, for any given iceberg costs, *lower* substitutability *increases* the *expected* value of the trade-to-output ratio common to both countries (see also the last but one column of panels I, II and III in Table 1). Another finding which stands out clearly in this graph is that as τ increases from 0 to 1 *(near-)linearity* of the peg trade share as a function of ν gradually transforms itself into a *steeper* and *more convex* curve.

For moderate to high shipment losses, lower substitutability also increases the *volatility* of trade shares across states of nature (cf. the standard deviation columns in each of panels II and III of Table 1). The intuition is that in this case consumers cannot substitute as much as they like to and would, *ceteris paribus*, so the resulting trade values measured *inclusive* of transport losses (that is, *cif*) inflate trade shares in output as defined for our purposes here and as already explained above.

Note, however, that for tiny costs of transport ($\tau = 0.01$) as in Panel I of Table 1 the relation in question appears not to be monotone. Instead, we have a divergence of simulated volatility away from 0 which corresponds to the widely-exploited in NOEM unit substitutability special case, $\nu = 1$, into higher magnitudes in both directions: $\nu \to \infty$ and $\nu \to 0$.

6 Concluding Comments

The model we developed here is useful to study the role of monetary uncertainty under trade frictions and import demand elasticity in determining the effects

Home and $\frac{c_{F,s}+c^*_{H,s}}{c^*_{F,s}+\frac{c^*_{F,s}}{1-\tau}}$ and for Foreign, because one cannot add up "apples to oranges".

[36] When tiny transport frictions ($\tau = 0.01$) are allowed for as well (see Panel I of Table 1), there is, however, no monotone function describing the relation discussed here, so trade variability generally depends on the particular parameter constellation.

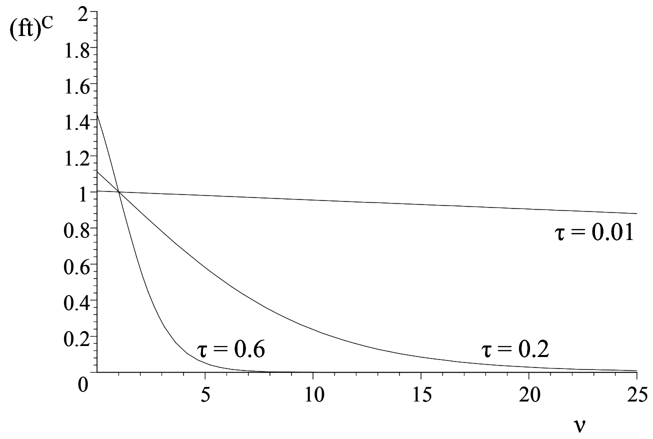

Figure 5: Peg Trade Share Curves across Substitutabilities for Given Iceberg Costs

of the exchange-rate regime on trade relative to output. Given symmetry of structures and shocks, we distinguished *two types* of effects, namely an *expected level* (or *first*-order) effect and a *volatility* (or *second*-order) effect.

Expected Level Effects Our unified NOEM framework designed to nest trade in both similar and different output mixes has clearly indicated when a peg would increase *expected* trade, the relevant measure in a stochastic setting as ours here, relative to a float and when not. A peg increases expected trade-to-output under *inelastic* import demand for a different foreign good-type valued equally as the domestic one and decreases it under elastic demand for similar composites produced in various brands across two symmetric economies. We have also concluded that this effect, although qualitatively novel and important, is quantitatively very *weak*, as already established in other models within the NOEM literature examining welfare issues. By contrast, a *strong* effect on the magnitude of the expected trade share in GDP has been found for some deeper trade "fundamentals" such as transport or tariff frictions and cross-country good-type substitutability, which is another contribution of the present paper. More precisely, such real determinants affect – via the optimally arising home bias – both the expected level of trade-to-output and its volatility across states, in a different way under elastic vs. inelastic import demand. Some of these trade fundamentals can relatively quickly be affected by policy (e.g. tariffs). Changing the structural underpinnings of other would, by contrast, take more time (e.g. transportation technologies or preferences).

Variability Effects What fixing the exchange rate can achieve under *PCP* (but not CCP) – by shutting down the pass-through and expenditure-switching channel – is to stabilize and equalize national trade shares across states of nature at their expected level. Our simulations have indicated that *how much* trade stabilization would be achieved by a passage from a fixed to a flexible exchange-rate regime ultimately depends on both monetary and real trade determinants. Within the perspective of actual-world economies and as a lesson for policy, the *degree* of trade share variability thus eliminated would be greater for (symmetric) nations, or currency unions, which (i) have a larger proportion of PCP in their (bilateral) trade, (ii) are exposed to higher monetary uncertainty and – for moderate to high costs of international exchange – (iii) produce less substitutable outputs and (iv) are located closer to one another or apply weaker (reciprocal) tariff and non-tariff restrictions. Therefore, the lesser the extent to which these conditions are met, the less efficient would a peg be as an instrument in stabilizing trade, simultaneously equalizing the ex-post utility of the ex-ante identical residents.

A Optimization and Equilibrium

A.1 Households Optimization Problem

The details of households *consumption basket* optimization in each realized state of nature s differ across our price-setting assumptions, so we present in turn the CCP and the PCP cases.

CCP Optimization of *Home* Households Under CCP, a H household's $j \in [0,1]$ *total real consumption demand* is defined by a Dixit-Stiglitz (1977) aggregator of the following form:

$$c_s^{j,C} \equiv \left[\left(\tfrac{1}{2}\right)^{\frac{1}{\nu}} \left(c_{H,s}^{j,C}\right)^{\frac{\nu-1}{\nu}} + \left(\tfrac{1}{2}\right)^{\frac{1}{\nu}} \left(c_{F,s}^{j,C}\right)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \quad \forall s \in S$$

Standard *representative* household's *cost minimization* ex-post, i.e. under certainty for any realized state of nature s, then progressively derives the expressions reported in the summary tables below.

CCP: H ***Domestic Absorption*** **(**$c_{i,s}^{j,C} \to c_{H,s}^{C}$**) and *PPI* (**$P_i^C \to P_H^C$**) Aggregation**

CCP Summary Table 1H

	$c_{i,s}^{j,C}, P_i^C, \quad j \in [0,1], \quad i \in [0,1], \quad \forall s \in S$
(1_H^H)	$c_{H,s}^{j,C} \equiv \left[\int_0^1 \left(c_{i,s}^{j,C}\right)^{\frac{\varphi-1}{\varphi}} di \right]^{\frac{\varphi}{\varphi-1}}$ by *index* definition
(2_H^H)	P_i^C given (preset in HC by a H firm i) $\Leftrightarrow$ state-independent
$(3_H^H C)$	$c_{i,s}^{j,C} = \left(\frac{P_i^C}{P_H^C}\right)^{-\varphi} c_{H,s}^{j,C} \Rightarrow c_{i,s}^{C} = \left(\frac{P_i^C}{P_H^C}\right)^{-\varphi} c_{H,s}^{C}$
(4_H^H)	$P_H^C \equiv \left[\int_0^1 \left(P_i^C\right)^{1-\varphi} di \right]^{\frac{1}{1-\varphi}}$ defined as the price of a *unit* of $c_{H,s}^C$
$(5_H^H C)$	$c_{H,s}^{j,C} = \frac{1}{2} \left(\frac{P_H^C}{P^C}\right)^{-\nu} \frac{M_s^j}{P^C} \Rightarrow c_{H,s}^{C} = \frac{1}{2} \left(\frac{P_H^C}{P^C}\right)^{-\nu} \frac{M_s}{P^C}$
$(3a_H^H C)$	$c_{i,s}^{j,C} = \frac{1}{2} \left(\frac{P_i^C}{P_H^C}\right)^{-\varphi} \left(\frac{P_H^C}{P^C}\right)^{-\nu} \frac{M_s^j}{P^C} \Rightarrow c_{i,s}^{C} = \frac{1}{2} \left(\frac{P_i^C}{P_H^C}\right)^{-\varphi} \left(\frac{P_H^C}{P^C}\right)^{-\nu} \frac{M_s}{P^C}$

CCP: H ***Import Demand*** $\left(c^{j,C}_{i^*,s} \to c^{C}_{F,s}\right)$ **and** ***Import Price Index*** $\left(P^C_{i^*} \to P^C_F\right)$ **Aggregation**

CCP SUMMARY TABLE 2H

	$c^{j,C}_{i^*,s}, P^C_{i^*}, \quad j \in [0,1], \quad i^* \in [1,2], \quad \forall s \in S$
$\left(1^H_F\right)$	$c^{j,C}_{F,s} \equiv \left[\int_1^2 \left(c^{j,C}_{i^*,s}\right)^{\frac{\varphi-1}{\varphi}} di^*\right]^{\frac{\varphi}{\varphi-1}}$ by *index* definition
$\left(2^H_F C\right)$	$P^C_{i^*}$ given (preset in HC by a F firm i^*) $\Leftrightarrow$ state-independent
$\left(3^H_F C\right)$	$c^{j,C}_{i^*,s} = \left(\frac{P^C_{i^*}}{P^C_F}\right)^{-\varphi} c^{j,C}_{F,s} \Rightarrow c^{C}_{i^*,s} = \left(\frac{P^C_{i^*}}{P^C_F}\right)^{-\varphi} c^{C}_{F,s}$
$\left(4^H_F C\right)$	$P^C_F \equiv \left[\int_1^2 \left(P^C_{i^*}\right)^{1-\varphi} di^*\right]^{\frac{1}{1-\varphi}}$ defined as the price of a *unit* of $c^C_{F,s}$
$\left(5^H_F C\right)$	$c^{j,C}_{F,s} = \frac{1}{2}\left(\frac{P^C_F}{P^C}\right)^{-\nu} \frac{M^j_s}{P^C} \Rightarrow c^{C}_{F,s} = \frac{1}{2}\left(\frac{P^C_F}{P^C}\right)^{-\nu} \frac{M_s}{P^C}$
$\left(3a^H_F C\right)$	$c^{j,C}_{i^*,s} = \frac{1}{2}\left(\frac{P^C_{i^*}}{P^C_F}\right)^{-\varphi}\left(\frac{P^C_F}{P^C}\right)^{-\nu} \frac{M^j_s}{P^C} \Rightarrow c^{C}_{i^*,s} = \frac{1}{2}\left(\frac{P^C_{i^*}}{P^C_F}\right)^{-\varphi}\left(\frac{P^C_F}{P^C}\right)^{-\nu} \frac{M_s}{P^C}$

CCP: H ***CPI*** $\left(P^C\right)$ **Aggregation**

CCP SUMMARY TABLE 3H

	$P^C, \quad i \in [0,1] \cup i^* \in [1,2], \quad \forall s \in S$
$\left(6^H C\right)$	$P^C \equiv \left[\frac{1}{2}\left(P^C_H\right)^{1-\nu} + \frac{1}{2}\left(P^C_F\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}$
$\left(6a^H C\right)$	$P^C \equiv \left(\frac{1}{2}\left\{\left[\int_0^1 \left(P^C_i\right)^{1-\varphi} di\right]^{\frac{1}{1-\varphi}}\right\}^{1-\nu} + \frac{1}{2}\left\{\left[\int_1^2 \left(P^C_{i^*}\right)^{1-\varphi} di^*\right]^{\frac{1}{1-\varphi}}\right\}^{1-\nu}\right)^{\frac{1}{1-\nu}}$

CCP Optimization of *Foreign* Households Under CCP, a F household's $j^* \in (1,2]$ *total real consumption demand* is analogously (or symmetrically) defined by the Dixit-Stiglitz (1977) aggregator:

$$c^{j^*,C}_s \equiv \left[\left(\tfrac{1}{2}\right)^{\frac{1}{\nu}}\left(c^{j^*,C}_{F,s}\right)^{\frac{\nu-1}{\nu}} + \left(\tfrac{1}{2}\right)^{\frac{1}{\nu}}\left(c^{j^*,C}_{H,s}\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \quad \forall s \in S$$

Standard *representative* household's *cost minimization* under certainty, i.e. for any realized state of nature s, then progressively derives the expressions below for Foreign that *parallel* (or, more precisely, are the mirror image of) those for Home.

CCP: F ***Domestic Absorption*** $(c_{i^*,s}^{j*,C} \to c_{F,s}^{*,C})$ **and** ***PPI*** $(P_{i^*}^{*,C} \to P_{F}^{*,C})$ **Aggregation**

CCP SUMMARY TABLE 1F

$c_{i^*,s}^{j^*,C}, P_{i^*}^{*,C}, \quad j^* \in [1,2], \quad i^* \in [1,2], \quad \forall s \in S$	
(1_F^F)	$c_{F,s}^{j^*,C} \equiv \left[\int_1^2 \left(c_{i^*,s}^{j^*,C} \right)^{\frac{\varphi^*-1}{\varphi^*}} di^* \right]^{\frac{\varphi^*}{\varphi^*-1}}$ by *index* definition
(2_F^F)	$P_{i^*}^{*,C}$ given (preset in FC by a F firm i^*) $\Leftrightarrow$ state-independent
$(3_F^F C)$	$c_{i^*,s}^{j^*,C} = \left(\frac{P_{i^*}^{*,C}}{P_F^{*,C}} \right)^{-\varphi^*} c_{F,s}^{j^*,C} \Rightarrow c_{i^*,s}^{*,C} = \left(\frac{P_{i^*}^{*,C}}{P_F^{*,C}} \right)^{-\varphi^*} c_{F,s}^{*,C}$
(4_F^F)	$P_F^{*,C} \equiv \left[\int_1^2 \left(P_{i^*}^{*,C} \right)^{1-\varphi^*} di^* \right]^{\frac{1}{1-\varphi^*}}$ defined as the price of a *unit* of $c_{F,s}^{*,C}$
(5_F^F)	$c_{F,s}^{j^*,C} = \frac{1}{2} \left(\frac{P_F^{*,C}}{P^{*,C}} \right)^{-\nu} \frac{M_s^{*,j^*}}{P^{*,C}} \Rightarrow c_{F,s}^{*,C} = \frac{1}{2} \left(\frac{P_F^{*,C}}{P^{*,C}} \right)^{-\nu} \frac{M_s^*}{P^{*,C}}$
$(3a_F^F)$	$c_{i^*,s}^{j^*,C} = \frac{1}{2} \left(\frac{P_{i^*}^{*,C}}{P_F^{*,C}} \right)^{-\varphi^*} \left(\frac{P_F^{*,C}}{P^{*,C}} \right)^{-\nu} \frac{M_s^{*,j^*}}{P^{*,C}} \Rightarrow c_{i^*,s}^{*,C} = \frac{1}{2} \left(\frac{P_{i^*}^{*,C}}{P_F^{*,C}} \right)^{-\varphi^*} \left(\frac{P_F^{*,C}}{P^{*,C}} \right)^{-\nu} \frac{M_s^*}{P^{*,C}}$

CCP: F **Import Demand** $(c_{i,s}^{j^*,C} \to c_{H,s}^{*,C})$ **and Import Price Index** $(P_i^{*,C} \to P_H^{*,C})$ **Aggregation**

CCP SUMMARY TABLE 2F

$c_{i,s}^{j^*,C}, P_i^{*,C}, \quad j^* \in [1,2], \quad i \in [0,1], \quad \forall s \in S$	
(1_H^F)	$c_{H,s}^{j^*,C} \equiv \left[\int_0^1 \left(c_{i,s}^{j^*,C} \right)^{\frac{\varphi^*-1}{\varphi^*}} di \right]^{\frac{\varphi^*}{\varphi^*-1}}$ by *index* definition
$(2_H^F C)$	$P_i^{*,C}$ given (preset in FC by a H firm i) $\Leftrightarrow$ state independent
$(3_H^F C)$	$c_{i,s}^{j^*,C} = \left(\frac{P_i^{*,C}}{P_H^{*,C}} \right)^{-\varphi^*} c_{H,s}^{j^*,C} \Rightarrow c_{i,s}^{C} = \left(\frac{P_i^{*,C}}{P_H^{*,C}} \right)^{-\varphi^*} c_{H,s}^{*,C}$
$(4_H^F C)$	$P_H^{*,C} \equiv \left[\int_0^1 \left(P_i^{*,C} \right)^{1-\varphi^*} di \right]^{\frac{1}{1-\varphi^*}}$ defined as the price of a *unit* of $c_{H,s}^{*,C}$
$(5_H^F C)$	$c_{H,s}^{j^*,C} = \frac{1}{2} \left(\frac{P_H^{*,C}}{P^{*,C}} \right)^{-\nu} \frac{M_s^{*,j^*}}{P^{*,C}} \Rightarrow c_{H,s}^{*,C} = \frac{1}{2} \left(\frac{P_H^{*,C}}{P^{*,C}} \right)^{-\nu} \frac{M_s^*}{P^{*,C}}$
$(3a_H^F C)$	$c_{i,s}^{j^*,C} = \frac{1}{2} \left(\frac{P_i^{*,C}}{P_H^{*,C}} \right)^{-\varphi^*} \left(\frac{P_H^{*,C}}{P^{*,C}} \right)^{-\nu} \frac{M_s^{*,j^*}}{P^{*,C}} \Rightarrow c_{i,s}^{*,C} = \frac{1}{2} \left(\frac{P_i^{*,C}}{P_H^{*,C}} \right)^{-\varphi^*} \left(\frac{P_H^{*,C}}{P^{*,C}} \right)^{-\nu} \frac{M_s^*}{P^{*,C}}$

CCP: F ***CPI*** $\left(P^{*,C}\right)$ **Aggregation**

CCP Summary Table 3F

$P^{*,C}, \quad i^{*} \in [1,2] \cup i \in [0,1], \quad \forall s \in S$	
$\left(6^{F}C\right)$	$P^{*,C} \equiv \left[\frac{1}{2}\left(P_{F}^{*,C}\right)^{1-\nu} + \frac{1}{2}\left(P_{H}^{*,C}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}$
$\left(6a^{F}C\right)$	$P^{*,C} \equiv \left(\frac{1}{2}\left\{\left[\int_{1}^{2}\left(P_{i^{*}}^{*,C}\right)^{1-\varphi^{*}} di^{*}\right]^{\frac{1}{1-\varphi^{*}}}\right\}^{1-\nu} + \frac{1}{2}\left\{\left[\int_{0}^{1}\left(P_{i}^{*,C}\right)^{1-\varphi^{*}} di\right]^{\frac{1}{1-\varphi^{*}}}\right\}^{1-\nu}\right)^{\frac{1}{1-\nu}}$

PCP Optimization of *Home* Households Under PCP, a H household's $j \in [0,1]$ *total real consumption demand* is again defined by a Dixit-Stiglitz (1977) index of the same form as under CCP but with different resulting domestic and external demands for goods (hence the P superscript indexing for PCP now in place of the C superscript indexing for CCP earlier) :

$$c_{s}^{j,P} \equiv \left[\left(\tfrac{1}{2}\right)^{\frac{1}{\nu}}\left(c_{H,s}^{j,P}\right)^{\frac{\nu-1}{\nu}} + \left(\tfrac{1}{2}\right)^{\frac{1}{\nu}}\left(c_{F,s}^{j,P}\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \quad \forall s \in S$$

Standard representative household's *cost minimization* ex-post, i.e. for any realized state of nature s, derives again the expressions in the summary tables below that parallel those reported for the CCP model version.

PCP: H ***Domestic Absorption*** $(c_{i,s}^{j,P} \rightarrow c_{H,s}^{P})$ **and** ***PPI*** $(P_{i}^{P} \rightarrow P_{H}^{P})$ **Aggregation**

PCP Summary Table 1H

$c_{i,s}^{j,P}, P_{i,s}^{P}, \quad j \in [0,1], \quad i \in [0,1], \quad \forall s \in S$	
$\left(1_{H}^{H}\right)$	$c_{H,s}^{j,P} \equiv \left[\int_{0}^{1}\left(c_{i,s}^{j,P}\right)^{\frac{\varphi-1}{\varphi}} di\right]^{\frac{\varphi}{\varphi-1}}$ by *index* definition
$\left(2_{H}^{H}\right)$	P_{i}^{P} given (preset in HC by a H firm i) $\Leftrightarrow$ state-independent
$\left(3_{H}^{H}P\right)$	$c_{i,s}^{j,P} = \left(\frac{P_{i}^{P}}{P_{H}^{P}}\right)^{-\varphi} c_{H,s}^{j,P} \Rightarrow c_{i,s}^{P} = \left(\frac{P_{i}^{P}}{P_{H}^{P}}\right)^{-\varphi} c_{H,s}^{P}$
$\left(4_{H}^{H}\right)$	$P_{H}^{P} \equiv \left[\int_{0}^{1}\left(P_{i}^{P}\right)^{1-\varphi} di\right]^{\frac{1}{1-\varphi}}$ defined as the price of a *unit* of $c_{H,s}^{P}$
$\left(5_{H}^{H}P\right)$	$c_{H,s}^{j,P} = \frac{1}{2}\left(\frac{P_{H}^{P}}{P_{s}^{P}}\right)^{-\nu}\frac{M_{s}^{j}}{P_{s}^{P}} \Rightarrow c_{H,s}^{P} = \frac{1}{2}\left(\frac{P_{H}^{P}}{P_{s}^{P}}\right)^{-\nu}\frac{M_{s}}{P_{s}^{P}}$
$\left(3a_{H}^{H}P\right)$	$c_{i,s}^{j,P} = \frac{1}{2}\left(\frac{P_{i}^{P}}{P_{H}^{P}}\right)^{-\varphi}\left(\frac{P_{H}^{P}}{P_{s}^{P}}\right)^{-\nu}\frac{M_{s}^{j}}{P_{s}^{P}} \Rightarrow c_{i,s}^{P} = \frac{1}{2}\left(\frac{P_{i}^{P}}{P_{H}^{P}}\right)^{-\varphi}\left(\frac{P_{H}^{P}}{P_{s}^{P}}\right)^{-\nu}\frac{M_{s}}{P_{s}^{P}}$

PCP: H ***Import Demand*** $(c^{j,P}_{i^*,s} \rightarrow c^{P}_{F,s})$ **and** ***Import Price Index*** $(P^{*,P}_{i^*} \overset{S^P_s}{\rightarrow} P^{P}_{i^*,s} \rightarrow P^{P}_{F,s})$ **Aggregation**

PCP Summary Table 2H

	$c^{j,P}_{i^*,s}, P^{*,P}_{i^*} \overset{S^P_s}{\rightarrow} P^{P}_{i^*,s}, \quad j \in [0,1], \quad i^* \in [1,2], \quad \forall s \in S$
(1^H_F)	$c^{j,P}_{F,s} \equiv \left[\int_1^2 \left(c^{j,P}_{i^*,s} \right)^{\frac{\varphi-1}{\varphi}} di^* \right]^{\frac{\varphi}{\varphi-1}}$ by *index* definition
$(2^H_F P)$	$P^{*,P}_{i^*}$ given (preset in FC by a F firm i^*) $\Leftrightarrow$ state independent
$(3^H_F P)$	$c^{j,P}_{i^*,s} = \left(\frac{P^{*,P}_{i^*}}{P^{*,P}_{F}} \right)^{-\varphi} c^{j,P}_{F,s} \Rightarrow c^{P}_{i^*,s} = \left(\frac{P^{*,P}_{i^*}}{P^{*,P}_{F}} \right)^{-\varphi} c^{P}_{F,s}$
$(4^H_F P)$	$\underbrace{\frac{S^P_s P^{*,P}_F}{1-\tau}}_{\equiv P^P_{F,s}} \equiv \left[\int_1^2 \underbrace{\left(\frac{S^P_s P^{*,P}_{i^*}}{1-\tau} \right)^{1-\varphi}}_{\equiv P^P_{i^*,s}} di^* \right]^{\frac{1}{1-\varphi}}$ defined as the price of a *unit* of $c^P_{F,s}$
$(5^H_F P)$	$c^{j,P}_{F,s} = \frac{1}{2} \left(\frac{\overbrace{\frac{S^P_s P^{*,P}_F}{1-\tau}}^{\equiv P^P_{F,s}}}{P^P_s} \right)^{-\nu} \frac{M^j_s}{P^P_s} \Rightarrow c^{P}_{F,s} = \frac{1}{2} \left(\frac{\overbrace{\frac{S^P_s P^{*,P}_F}{1-\tau}}^{\equiv P^P_{F,s}}}{P^P_s} \right)^{-\nu} \frac{M_s}{P^P_s}$
$(3a^H_F P)$	$c^{j,P}_{i^*,s} = \frac{1}{2} \left(\frac{P^{*,P}_{i^*}}{P^{*,P}_{F}} \right)^{-\varphi} \left(\frac{\overbrace{\frac{S^P_s P^{*,P}_F}{1-\tau}}^{\equiv P^P_{F,s}}}{P^P_s} \right)^{-\nu} \frac{M^j_s}{P^P_s} \Rightarrow c^{P}_{i^*,s} = \frac{1}{2} \left(\frac{P^{*,P}_{i^*}}{P^{*,P}_{F}} \right)^{-\varphi} \left(\frac{\overbrace{\frac{S^P_s P^{*,P}_F}{1-\tau}}^{\equiv P^P_{F,s}}}{P^P_s} \right)^{-\nu} \frac{M_s}{P^P_s}$

PCP: H *CPI* $\left(P_s^P\right)$ Aggregation

PCP Summary Table 3H

	$P_s^P, \quad i \in [0,1] \cup i^* \in [1,2], \quad \forall s \in S$
$(6^H P)$	$P_s^P \equiv \left[\frac{1}{2}\left(P_H^P\right)^{1-\nu} + \frac{1}{2}\left(P_{F,s}^P\right)^{1-\nu}\right]^{\frac{1}{1-\nu}} \equiv \left[\frac{1}{2}\left(P_H^P\right)^{1-\nu} + \frac{1}{2}\left(\frac{S_s^P P_F^{*,P}}{1-\tau}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}$
$(6a^H P)$	$P_s^P \equiv \left(\frac{1}{2}\left\{\left[\int_0^1 \left(P_i^P\right)^{1-\varphi} di\right]^{\frac{1}{1-\varphi}}\right\}^{1-\nu} + \frac{1}{2}\left\{\left[\int_1^2 \left(P_{i^*,s}^P\right)^{1-\varphi} di^*\right]^{\frac{1}{1-\varphi}}\right\}^{1-\nu}\right)^{\frac{1}{1-\nu}} \equiv \left(\frac{1}{2}\left\{\left[\int_0^1 \left(P_i^P\right)^{1-\varphi} di\right]^{\frac{1}{1-\varphi}}\right\}^{1-\nu} + \frac{1}{2}\left\{\left[\int_1^2 \left(\frac{S_s^P P_{i^*}^{*,P}}{1-\tau}\right)^{1-\varphi} di^*\right]^{\frac{1}{1-\varphi}}\right\}^{1-\nu}\right)^{\frac{1}{1-\nu}}$

PCP Optimization of *Foreign* Households Under PCP, a F household's $j^* \in (1,2]$ *total real consumption demand* is defined, analogously to that of a H household $j \in [0,1]$, by a Dixit-Stiglitz (1977) index of the same form:

$$c_s^{j^*,P} \equiv \left[\left(\tfrac{1}{2}\right)^{\frac{1}{\nu}}\left(c_{F,s}^{j^*,P}\right)^{\frac{\nu-1}{\nu}} + \left(\tfrac{1}{2}\right)^{\frac{1}{\nu}}\left(c_{H,s}^{j^*,P}\right)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \quad \forall s \in S$$

Standard *representative* household's *cost minimization* under certainty, i.e. for any realized state of nature s, then derives the expressions in the summary tables that follow.

PCP: F ***Domestic Absorption*** **($c_{i^*,s}^{j^*,P} \to c_{F,s}^{*,P}$) and *PPI* ($P_{i^*}^{*,P} \to P_F^{*,P}$) Aggregation**

PCP Summary Table 1F

	$c_{i^*,s}^{j^*,P}, P_{i^*}^{*,P}, \quad j^* \in [1,2], \quad i^* \in [1,2], \quad \forall s \in S$
(1_F^F)	$c_{F,s}^{j^*,P} \equiv \left[\int_1^2 \left(c_{i^*,s}^{j^*,P}\right)^{\frac{\varphi^*-1}{\varphi^*}} di^*\right]^{\frac{\varphi^*}{\varphi^*-1}}$ by *index* definition
(2_F^F)	$P_{i^*}^{*,P}$ given (preset in FC by a F firm i^*) $\Leftrightarrow$ state-independent
$(3_F^F P)$	$c_{i^*,s}^{j^*,P} = \left(\frac{P_{i^*}^{*,P}}{P_F^{*,P}}\right)^{-\varphi^*} c_{F,s}^{j^*,P} \Rightarrow c_{i^*,s}^{*,P} = \left(\frac{P_{i^*}^{*,P}}{P_F^{*,P}}\right)^{-\varphi^*} c_{F,s}^{*,P}$
(4_F^F)	$P_F^{*,P} \equiv \left[\int_1^2 \left(P_{i^*}^{*,P}\right)^{1-\varphi^*} di^*\right]^{\frac{1}{1-\varphi^*}}$ defined as the price of a *unit* of $c_{F,s}^{*,P}$
$(5_F^F P)$	$c_{F,s}^{j^*,P} = \frac{1}{2}\left(\frac{P_F^{*,P}}{P_s^{*,P}}\right)^{-\nu} \frac{M_s^{*,j^*}}{P_s^{*,P}} \Rightarrow c_{F,s}^{*,P} = \frac{1}{2}\left(\frac{P_F^{*,P}}{P_s^{*,P}}\right)^{-\nu} \frac{M_s^*}{P_s^{*,P}}$
$(3a_F^F P)$	$c_{i^*,s}^{j^*,P} = \frac{1}{2}\left(\frac{P_{i^*}^{*,P}}{P_F^{*,P}}\right)^{-\varphi^*}\left(\frac{P_F^{*,P}}{P_s^{*,P}}\right)^{-\nu} \frac{M_s^{*,j^*}}{P_s^{*,P}} \Rightarrow c_{i^*,s}^{*,P} = \frac{1}{2}\left(\frac{P_{i^*}^{*,P}}{P_F^{*,P}}\right)^{-\varphi^*}\left(\frac{P_F^{*,P}}{P_s^{*,P}}\right)^{-\nu} \frac{M_s^*}{P_s^{*,P}}$

PCP: F ***Import Demand*** $\left(c_{i,s}^{j^*,P} \rightarrow c_{H,s}^{*,P}\right)$ **and** ***Import Price Index*** $\left(P_i^P \overset{S_s^P}{\rightarrow} P_{i,s}^{*,P} \rightarrow P_{H,s}^{*,P}\right)$ **Aggregation**

PCP SUMMARY TABLE 2F

	$c_{i,s}^{j^*,P}, P_i^P \overset{S_s^P}{\rightarrow} P_{i,s}^{*,P}, \quad j^* \in [1,2], \quad i \in [0,1], \quad \forall s \in S$
$\left(1_H^F\right)$	$c_{H,s}^{j^*,P} \equiv \left[\int_0^1 \left(c_{i,s}^{j^*,P}\right)^{\frac{\varphi^*-1}{\varphi^*}} di\right]^{\frac{\varphi^*}{\varphi^*-1}}$ by *index* definition
$\left(2_H^F P\right)$	P_i^P given (preset in HC by a H firm i) $\Leftrightarrow$ state-independent
$\left(3_H^F P\right)$	$c_{i,s}^{j^*,P} = \left(\frac{P_i^P}{P_H^P}\right)^{-\varphi^*} c_{H,s}^{j^*,P} \Rightarrow c_{i,s}^{*,P} = \left(\frac{P_i^P}{P_H^P}\right)^{-\varphi^*} c_{H,s}^{*,P}$
$\left(4_H^F P\right)$	$\underbrace{\frac{P_H}{S_s^P\left(1-\tau\right)}}_{\equiv P_{H,s}^{*,P}} \equiv \left[\int_0^1 \left(\underbrace{\frac{P_i}{S_s^P\left(1-\tau\right)}}_{\equiv P_{i,s}^{*,P}}\right)^{1-\varphi^*} di\right]^{\frac{1}{1-\varphi^*}}$ defined as the price of a *unit* of $c_{H,s}^{*,P}$
$\left(5_H^F P\right)$	$c_{H,s}^{j^*,P} = \frac{1}{2}\left(\frac{\overbrace{\frac{P_H^P}{S_s^P(1-\tau)}}^{\equiv P_{H,s}^{*,P}}}{P_s^{*,P}}\right)^{-\nu} \frac{M_s^{*,j^*}}{P_s^{*,P}} \Rightarrow c_{H,s}^{*,P} = \frac{1}{2}\left(\frac{\overbrace{\frac{P_H^P}{S_s^P(1-\tau)}}^{\equiv P_{H,s}^{*,P}}}{P_s^{*,P}}\right)^{-\nu} \frac{M_s^{*}}{P_s^{*,P}}$
$\left(3a_H^F P\right)$	$c_{i,s}^{j^*,P} = \frac{1}{2}\left(\frac{P_i^P}{P_H^P}\right)^{-\varphi^*}\left(\frac{\overbrace{\frac{P_H^P}{S_s^P(1-\tau)}}^{\equiv P_{H,s}^{*,P}}}{P_s^{*,P}}\right)^{-\nu} \frac{M_s^{*,j^*}}{P_s^{*,P}} \Rightarrow c_{i,s}^{P} = \frac{1}{2}\left(\frac{P_i^P}{P_H^P}\right)^{-\varphi^*}\left(\frac{\overbrace{\frac{P_H^P}{S_s^P(1-\tau)}}^{\equiv P_{H,s}^{*,P}}}{P_s^{*,P}}\right)^{-\nu} \frac{M_s^{*}}{P_s^{*,P}}$

PCP: F ***CPI*** $\left(P_s^{*,P}\right)$ **Aggregation**

PCP SUMMARY TABLE 3F

	$P_s^{*,P}, \quad i^* \in [1,2] \cup i \in [0,1], \quad \forall s \in S$
$\left(6^F P\right)$	$P_s^{*,P} \equiv \left[\frac{1}{2}\left(P_F^{*,P}\right)^{1-\nu} + \frac{1}{2}\left(P_{H,s}^{*,P}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}} \equiv$
	$\equiv \left[\frac{1}{2}\left(P_F^{*,P}\right)^{1-\nu} + \frac{1}{2}\left(\frac{P_H^P}{S_s^P(1-\tau)}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}$
$\left(6a^F P\right)$	$P_s^{*,P} \equiv \left(\frac{1}{2}\left\{\left[\int_0^1 \left(P_{i^*}^{*,P}\right)^{1-\varphi^*} di^*\right]^{\frac{1}{1-\varphi^*}}\right\}^{1-\nu} + \frac{1}{2}\left\{\left[\int_1^2 \left(P_{i,s}^{*,P}\right)^{1-\varphi^*} di\right]^{\frac{1}{1-\varphi^*}}\right\}^{1-\nu}\right)^{\frac{1}{1-\nu}} \equiv$
	$\equiv \left(\frac{1}{2}\left\{\left[\int_0^1 \left(P_{i^*}^{*,P}\right)^{1-\varphi^*} di^*\right]^{\frac{1}{1-\varphi^*}}\right\}^{1-\nu} + \frac{1}{2}\left\{\left[\int_1^2 \left(\frac{P_i^P}{S_s^P(1-\tau)}\right)^{1-\varphi^*} di\right]^{\frac{1}{1-\varphi^*}}\right\}^{1-\nu}\right)^{\frac{1}{1-\nu}}$

A.2 Definition of Equilibrium

We now formally define an equilibrium concept that corresponds to the described sequential optimization.

Definition 1 *In the context of the model versions we presented, an* equilibrium *is a set of quantities and prices, such that:*

1. ***[Ex-Ante Conditions]*** before *the resolution of monetary uncertainty and given the commonly agreed upon joint symmetric distribution of money growth shocks* (μ_s, μ_s^*)*;*

   (a) *[Firms Stochastic Optimization] given their technology constraint and the* expected *quantities demanded in the* goods *market,* $\left\{E_0\left[c_{H,s}^C\right],\right.$ $E_0\left[c_{H,s}^{*,C}\right]$, $E_0\left[c_{F,s}^{*,C}\right]$, $\left.E_0\left[c_{F,s}^C\right]\right\}$ *under CCP or* $\left\{E_0\left[c_{H,s}^P\right], E_0\left[c_{H,s}^{*,P}\right],\right.$ $E_0\left[c_{F,s}^{*,P}\right]$, $\left.E_0\left[c_{F,s}^P\right]\right\}$ *under PCP, the prices,* $\left\{P_H^C, P_H^{*,C}, P_F^{*,C}, P_F^C\right\}$ *under CCP or* $\left\{P_H^P, P_F^{*,P}\right\}$ *under PCP, that are optimally preset ex-ante (i.e. in state* 0*) and bindingly posted to consumer households for transactions ex-post (in state* s *for* $\forall s \in S$*) solve the profit maximization problem of the representative producer firm in* Home *as well as in* Foreign*;*

2. ***[Ex-Post Conditions]*** following *the resolution of monetary uncertainty and in any state of nature* $s \in S$ *that has materialized;*

   (a) *[Households Labor-Leisure Trade-Off] given its constraints and the posted prices,* $\left\{P_H^C, P_H^{*,C}, P_F^{*,C}, P_F^C\right\}$ *under CCP or* $\left\{P_H^P, P_F^{*,P}\right\}$ *under PCP, the representative consumer household in* Home *as well as in* Foreign *spends up all available cash on its* total *real consumption* $\{c_s, c_s^*\}$*; hours of work (employment)* $\{1-l_s, 1-l_s^*\}$ *are supplied by households until firms demand labor to equilibrate ex-post consumption demand for their differentiated products at the resulting equilibrium real wage rates* $\left\{\dfrac{W_s}{P_s}, \dfrac{W_s^*}{P_s^*}\right\}$*;*

   (b) *[Households Consumer Basket Allocation] given the posted prices,* $\left\{P_H^C, P_H^{*,C}, P_F^{*,C}, P_F^C\right\}$ *under CCP or* $\left\{P_H^P, P_F^{*,P}\right\}$ *under PCP, the consumption quantities* $\left\{c_{H,s}^C, c_{H,s}^{*,C}, c_{F,s}^{*,C}, c_{F,s}^C\right\}$ *under CCP or* $\left\{c_{H,s}^P,\right.$ $\left.c_{H,s}^{*,P}, c_{F,s}^{*,P}, c_{F,s}^P\right\}$ *under PCP solve the cost minimization problem à la Dixit-Stiglitz (1977) of the representative consumer household in* Home *as well as in* Foreign*;*

   (c) *[Goods Market Clearing] all quantities under CCP or PCP satisfy the feasibility conditions for each* differentiated brand *so that all product-brand markets – and, hence, the* international *product-type market as a whole – clear;*

   (d) *[Forex Market Clearing] the* international *forex market clears as well.*

A.3 Equilibrium Nominal Exchange Rate

CCP Under CCP, the equilibrium (ex-post) NER solves the foreign exchange market clearing condition: excess supply of each of the two currencies (expressed in the same monetary unit) is zero in any state of nature s that has materialized. Taking the currency of H as the common unit of account, S_s^C is determined by:

$$\underbrace{P_F^C c_{F,s}^C}_{F \text{ export revenues} \Leftrightarrow HCsupply} \quad - \quad S_s^C \cdot \quad \underbrace{P_H^{*,C} c_{H,s}^{*,C}}_{H \text{ export revenues} \Leftrightarrow HCdemand} \quad = \quad 0$$

Substituting for optimal $c_{F,s}^C$ and $c_{H,s}^{*,C}$ above as well as for the ideal H and F CPI definitions further on in the algebraic manipulation derives:

$$P_F^C \underbrace{\frac{1}{2}\left(\frac{P_F^C}{P^C}\right)^{-\nu} \frac{M_s}{P^C}}_{=c_{F,s}^C} = S_s^C P_H^{*,C} \underbrace{\frac{1}{2}\left(\frac{P_H^{*,C}}{P^{*,C}}\right)^{-\nu} \frac{M_s^*}{P^{*,C}}}_{=c_{H,s}^{*,C}}$$

$$S_s^C = \frac{\left(\frac{P_F^C}{P^C}\right)^{1-\nu}}{\left(\frac{P_H^{*,C}}{P^{*,C}}\right)^{1-\nu}} \frac{M_s}{M_s^*}$$

$$S_s^C = \frac{\left(\frac{P_F^C}{\underbrace{\left[\left(\frac{1}{2}\left(P_H^C\right)^{1-\nu} + \frac{1}{2}\left(P_F^C\right)^{1-\nu}\right)^{\frac{1}{1-\nu}}\right]}_{\equiv P^C}}\right)^{1-\nu}}{\left(\frac{P_H^{*,C}}{\underbrace{\left[\left(\frac{1}{2}\left(P_F^{*,C}\right)^{1-\nu} + \frac{1}{2}\left(P_H^{*,C}\right)^{1-\nu}\right)^{\frac{1}{1-\nu}}\right]}_{\equiv P^{*,C}}}\right)^{1-\nu}} \frac{M_s}{M_s^*}$$

$$S_s^C = \frac{\frac{\left(P_F^C\right)^{1-\nu}}{\frac{1}{2}\left(P_H^C\right)^{1-\nu}+\frac{1}{2}\left(P_F^C\right)^{1-\nu}}}{\frac{\left(P_H^{*,C}\right)^{1-\nu}}{\frac{1}{2}\left(P_F^{*,C}\right)^{1-\nu}+\frac{1}{2}\left(P_H^{*,C}\right)^{1-\nu}}}\frac{M_s}{M_s^*} = \frac{\frac{\left(P_F^C\right)^{1-\nu}}{\frac{1}{2}\left[\left(P_H^C\right)^{1-\nu}+\left(P_F^C\right)^{1-\nu}\right]}}{\frac{\left(P_H^{*,C}\right)^{1-\nu}}{\frac{1}{2}\left[\left(P_F^{*,C}\right)^{1-\nu}+\left(P_H^{*,C}\right)^{1-\nu}\right]}}\frac{M_s}{M_s^*}$$

$$S_s^C = \frac{\frac{\left(P_F^C\right)^{1-\nu}}{\left(P_H^C\right)^{1-\nu}+\left(P_F^C\right)^{1-\nu}}}{\frac{\left(P_H^{*,C}\right)^{1-\nu}}{\left(P_F^{*,C}\right)^{1-\nu}+\left(P_H^{*,C}\right)^{1-\nu}}}\frac{M_s}{M_s^*} = \frac{\frac{1}{\frac{\left(P_H^C\right)^{1-\nu}+\left(P_F^C\right)^{1-\nu}}{\left(P_F^C\right)^{1-\nu}}}}{\frac{1}{\frac{\left(P_F^{*,C}\right)^{1-\nu}+\left(P_H^{*,C}\right)^{1-\nu}}{\left(P_H^{*,C}\right)^{1-\nu}}}}\frac{M_s}{M_s^*}$$

$$S_s^C = \frac{\frac{1}{\frac{\left(P_H^C\right)^{1-\nu}}{\left(P_F^C\right)^{1-\nu}}+1}}{\frac{1}{\frac{\left(P_F^{*,C}\right)^{1-\nu}}{\left(P_H^{*,C}\right)^{1-\nu}}+1}}\frac{M_s}{M_s^*} = \frac{1}{\frac{\left(P_H^C\right)^{1-\nu}}{\left(P_F^C\right)^{1-\nu}}+1}\frac{\frac{\left(P_F^{*,C}\right)^{1-\nu}}{\left(P_H^{*,C}\right)^{1-\nu}}+1}{1}\frac{M_s}{M_s^*}$$

$$S_s^C = \frac{1+\left(\frac{P_F^{*,C}}{P_H^{*,C}}\right)^{1-\nu}}{1+\left(\frac{P_H^C}{P_F^C}\right)^{1-\nu}+1}\frac{M_s}{M_s^*}$$

Now using the price equalities established earlier, namely $P_H = P_H^* = P_F^* = P_F^*$ to substitute above, one obtains:

$$S_s = \frac{M_s}{M_s^*}$$

which is the CCP expression in (21).

PCP Under PCP, the equilibrium (ex-post) NER solves, analogously to the CCP case, the foreign exchange market clearing condition. Taking again the monetary unit of H as the numéraire for the purposes of conversion, S_s^P is determined by:

$$S_s^P \cdot \underbrace{\frac{P_F^* c_{F,s}^P}{1-\tau}}_{F \text{ export revenues} \Leftrightarrow HCsupply} - \underbrace{\frac{P_H c_{H,s}^{*,P}}{1-\tau}}_{H \text{ export revenues} \Leftrightarrow HCdemand} = 0$$

$$S_s^P c_{F,s}^P = c_{H,s}^{*,P}$$

$$S_s^P \overbrace{\frac{1}{2}\left(\frac{P_{F,s}^P}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}}^{=c_{F,s}^P} = \overbrace{\frac{1}{2}\left(\frac{P_{H,s}^{*,P}}{P_s^{*,P}}\right)^{-\nu}\frac{M_s^*}{P_s^{*,P}}}^{=c_{H,s}^{*,P}}$$

$$S_s^P \left(\frac{\frac{S_s^P P_F^*}{1-\tau}}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P} = \left(\frac{\frac{P_H}{S_s^P(1-\tau)}}{P_s^{*,P}}\right)^{-\nu}\frac{M_s^*}{P_s^{*,P}}$$

$$S_s^P \frac{\left(\frac{\frac{S_s^P P_F^*}{1-\tau}}{P_s^P}\right)^{-\nu}}{\left(\frac{\frac{P_H}{S_s^P(1-\tau)}}{P_s^{*,P}}\right)^{-\nu}} = \frac{\frac{M_s^*}{P_s^{*,P}}}{\frac{M_s}{P_s^P}}$$

$$S_s^P \left(\frac{\frac{S_s^P P_F^*}{1-\tau}}{P_s^P}\frac{P_s^{*,P}}{\frac{P_H}{S_s^P(1-\tau)}}\right)^{-\nu} = \frac{M_s^*}{M_s}\frac{P_s^P}{P_s^{*,P}}$$

$$S_s^P \left(\frac{\frac{S_s^P}{1}}{P_s^P}\frac{P_s^{*,P}}{\frac{1}{S_s^P}}\right)^{-\nu} = \frac{M_s^*}{M_s}\frac{P_s^P}{P_s^{*,P}}$$

$$S_s^P \left(\left(S_s^P\right)^2\frac{P_s^{*,P}}{P_s^P}\right)^{-\nu} = \frac{M_s^*}{M_s}\frac{P_s^P}{P_s^{*,P}}$$

$$S_s^P \left(S_s^P\right)^{-2\nu}\left(\frac{P_s^{*,P}}{P_s^P}\right)^{-\nu} = \frac{M_s^*}{M_s}\frac{P_s^P}{P_s^{*,P}}$$

$$\left(S_s^P\right)^{1-2\nu}\left(\frac{P_s^P}{P_s^{*,P}}\right)^{\nu} = \frac{M_s^*}{M_s}\frac{P_s^P}{P_s^{*,P}}$$

$$\left(S_s^P\right)^{1-2\nu} = \frac{M_s^*}{M_s}\left(\frac{P_s^P}{P_s^{*,P}}\right)^{1-\nu}$$

$$S_s^P = \left(\frac{M_s^*}{M_s}\right)^{\frac{1}{1-2\nu}}\left(\frac{P_s^P}{P_s^{*,P}}\right)^{\frac{1-\nu}{1-2\nu}} \tag{33}$$

Now we use the CPI definitions derived earlier to substitute for their ratio above:

$$\frac{P_s^P}{P_s^{*,P}} = \frac{\left[\frac{1}{2}\left(P_H^P\right)^{1-\nu} + \frac{1}{2}\left(\frac{S_s^P P_F^{*,P}}{(1-\tau)}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}}{\left[\frac{1}{2}\left(P_F^{*,P}\right)^{1-\nu} + \frac{1}{2}\left(\frac{P_H^P}{S_s^P(1-\tau)}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}} =$$

$$= \left[\frac{\frac{1}{2}\left(P_H^P\right)^{1-\nu}\left(1+\frac{S_s^P}{1-\tau}\right)^{1-\nu}}{\frac{1}{2}\left(P_H^P\right)^{1-\nu}\left(1+\frac{1}{S_s^P(1-\tau)}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}} = \left[\frac{\left(1+\frac{S_s^P}{1-\tau}\right)^{1-\nu}}{\left(1+\frac{1}{S_s^P(1-\tau)}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}$$

So that:

$$S_s^P = \left(\frac{M_s^*}{M_s}\right)^{\frac{1}{1-2\nu}} \left(\frac{P_s^P}{P_s^{*,P}}\right)^{\frac{1-\nu}{1-2\nu}}$$

$$S_s^P = \left(\frac{M_s^*}{M_s}\right)^{\frac{1}{1-2\nu}} \left\{\left[\frac{\left(1+\frac{S_s^P}{1-\tau}\right)^{1-\nu}}{\left(1+\frac{1}{S_s^P(1-\tau)}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}\right\}^{\frac{1-\nu}{1-2\nu}}$$

$$\left(S_s^P\right)^{1-2\nu} = \frac{M_s^*}{M_s}\frac{\left(1+\frac{S_s^P}{1-\tau}\right)^{1-\nu}}{\left(1+\frac{1}{S_s^P(1-\tau)}\right)^{1-\nu}}$$

$$\left(S_s^P\right)^{1-2\nu} = \frac{M_s^*}{M_s}\left(1-\tau\right)^{1-\nu}\left(S_s^P\right)^{1-\nu}\frac{\left(1+\frac{S_s^P}{1-\tau}\right)^{1-\nu}}{1+\left(1-\tau\right)^{1-\nu}\left(S_s^P\right)^{1-\nu}}$$

$$\left(S_s^P\right)^{1-2\nu} = \frac{M_s^*}{M_s}\left(S_s^P\right)^{1-\nu}\frac{\left(1-\tau\right)^{1-\nu}+\left(S_s^P\right)^{1-\nu}}{1+\left(1-\tau\right)^{1-\nu}\left(S_s^P\right)^{1-\nu}}$$

$$\left(S_s^P\right)^{-\nu} = \frac{M_s^*}{M_s}\frac{\left(1-\tau\right)^{1-\nu}+\left(S_s^P\right)^{1-\nu}}{1+\left(1-\tau\right)^{1-\nu}\left(S_s^P\right)^{1-\nu}}$$

$$\left(S_s^P\right)^{\nu} = \frac{M_s}{M_s^*}\frac{1+\left(1-\tau\right)^{1-\nu}\left(S_s^P\right)^{1-\nu}}{\left(1-\tau\right)^{1-\nu}+\left(S_s^P\right)^{1-\nu}}$$

$$S_s^P = \left[\frac{1+\left(1-\tau\right)^{1-\nu}\left(S_s^P\right)^{1-\nu}}{\left(1-\tau\right)^{1-\nu}+\left(S_s^P\right)^{1-\nu}}\right]^{\frac{1}{\nu}}\left(\frac{M_s}{M_s^*}\right)^{\frac{1}{\nu}},$$

which is the PCP expression in (21).
Under a peg, i.e. with $M_s = M_s^*$ for any $s \in S$, one would further on obtain:

$$S_s^P = \left[\frac{1 + (1-\tau)^{1-\nu} \left(S_s^P\right)^{1-\nu}}{(1-\tau)^{1-\nu} + \left(S_s^P\right)^{1-\nu}} \right]^{\frac{1}{\nu}}$$

$$\left(S_s^P\right)^{\nu} = \frac{1 + (1-\tau)^{1-\nu} \left(S_s^P\right)^{1-\nu}}{(1-\tau)^{1-\nu} + \left(S_s^P\right)^{1-\nu}}$$

$$(1-\tau)^{1-\nu} \left(S_s^P\right)^{\nu} + \left(S_s^P\right)^{\nu} \left(S_s^P\right)^{1-\nu} = 1 + (1-\tau)^{1-\nu} \left(S_s^P\right)^{1-\nu}$$

$$(1-\tau)^{1-\nu} \left(S_s^P\right)^{\nu} + S_s^P = 1 + (1-\tau)^{1-\nu} \left(S_s^P\right)^{1-\nu}$$

$$(1-\tau)^{1-\nu} \left(S_s^P\right)^{\nu} - (1-\tau)^{1-\nu} \left(S_s^P\right)^{1-\nu} + S_s^P - 1 = 0,$$

which is (22) in the main text.

A.4 Equilibrium Trade Shares

With iceberg costs $0 < \tau < 1$ now taken into account, Home[37] CCP vs. PCP equilibrium trade/GDP ratio is defined by

$$(ft)_{H,s}^C \equiv \frac{(FT)_{H,s}^C}{Y_{H,s}^C} = \frac{(Ex)_{H,s}^{C,cif} + (Im)_{H,s}^{C,cif}}{(DA)_{H,s}^C + (Ex)_{H,s}^{C,cif}} = \frac{S_s^C \cdot \overbrace{P_H^{*,C}}^{\text{FC } cif} \cdot \overbrace{c_{H,s}^{*,C}}^{\text{consumed}} + \overbrace{P_F^C}^{\text{HC } cif} \cdot \overbrace{c_{F,s}^C}^{\text{consumed}}}{P_H^C \cdot c_{H,s}^C + S_s^C \cdot \underbrace{P_H^{*,C}}_{\text{FC } cif} \cdot \underbrace{c_{H,s}^{*,C}}_{\text{consumed}}} =$$

$$= \frac{S_s^C \cdot \overbrace{(1-\tau) P_H^{*,C}}^{\text{FC } fob} \cdot \overbrace{\frac{c_{H,s}^{*,C}}{1-\tau}}^{\text{produced}} + \overbrace{(1-\tau) P_F^C}^{\text{HC } fob} \cdot \overbrace{\frac{c_{F,s}^C}{1-\tau}}^{\text{produced}}}{P_H^C \cdot c_{H,s}^C + S_s^C \cdot \underbrace{(1-\tau) P_H^{*,C}}_{\text{FC } fob} \cdot \underbrace{\frac{c_{H,s}^{*,C}}{1-\tau}}_{\text{produced}}} \text{ vs.} \tag{34}$$

$$(ft)_{H,s}^P \equiv \frac{(FT)_{H,s}^P}{Y_{H,s}^P} = \frac{\overbrace{(Ex)_{H,s}^{P,fob}}^{\equiv (Ex)_{H,s}^{P,cif}} + \overbrace{(Im)_{H,s}^{P,fob}}^{\equiv (Im)_{H,s}^{P,cif}}}{(DA)_{H,s}^P + \underbrace{(Ex)_{H,s}^{P,fob}}_{\equiv (Ex)_{H,s}^{P,cif}}} = \frac{\overbrace{P_H^P}^{\text{HC } fob} \overbrace{\frac{c_{H,s}^{*,P}}{1-\tau}}^{\text{produced}} + S_s^P \overbrace{P_F^{*,P}}^{\text{FC } fob} \overbrace{\frac{c_{F,s}^P}{1-\tau}}^{\text{produced}}}{P_H^P c_{H,s}^P + \underbrace{P_H^P}_{\text{HC } fob} \underbrace{\frac{c_{H,s}^{*,P}}{1-\tau}}_{\text{produced}}}$$

$$= \frac{\overbrace{\frac{P_H^P}{1-\tau}}^{\text{HC } cif} \overbrace{c_{H,s}^{*,P}}^{\text{consumed}} + \overbrace{\frac{S_s^P P_F^{*,P}}{1-\tau}}^{\text{HC } cif} \overbrace{c_{F,s}^P}^{\text{consumed}}}{P_H^P c_{H,s}^P + \underbrace{\frac{P_H^P}{1-\tau}}_{\text{HC } cif} \underbrace{c_{H,s}^{*,P}}_{\text{consumed}}}, \tag{35}$$

[37] For Foreign, the respective expressions are symmetric.

where $(Ex)^{C,cif}_{H,s}$ denotes Home exports at *cif* prices, $(Im)^{C,cif}_{H,s}$ Home imports at *cif* prices and $(DA)^{C}_{H,s}$ Home domestic absorption, with all these three *values* (prices multiplied by quantities) expressed in Home currency under CCP for any state $s \in S$ that has materialized. $(Ex)^{P,fob}_{H,s}$, $(Im)^{P,fob}_{H,s}$ and $(DA)^{P}_{H,s}$ are, of course, the respective Home-currency values under PCP, with Home exports and imports now measured at *fob* prices. It is important to recall at this point that once a transport and/or tariff friction is considered in our extended NOEM model, the relevant prices for equilibrium trade flows as implied by the invoicing conventions we analyze become the *cif* ones under CCP and the *fob* ones under PCP. However, due to our *symmetric* iceberg costs assumption, we have shown by the last equalities in (34) and (35) above that the fob values are *exactly equal* to their respective cif values in both our CCP and PCP model versions, so that trade shares can be meaningfully compared across alternative price setting as if calculated on the *same*, *cif* basis. This latter, *cif domestic*-currency value is, furthermore, the appropriate measure to use, since it duly accounts for the difference between quantities *bought* and quantities *consumed* arising from the output lost in transit and thus reflects the *true cost* to the representative consumer.

Substitutions for optimal domestic and external demands for H and F output and use of the ideal CPI definitions derive – under *full symmetry* and *separable preferences* – the CCP vs. PCP equilibrium trade shares in the main text.

The derivation under CCP for Home is:

$$(ft)^{C}_{H,s} \equiv \frac{(FT)^{C}_{H,s}}{Y^{C}_{H,s}} = \frac{(Ex)^{C,cif}_{H,s}+(Im)^{C,cif}_{H,s}}{(DA)^{C}_{H,s}+(Ex)^{C,cif}_{H,s}} = \frac{S^{C}_{s}\cdot \overbrace{P^{*,C}_{H}}^{\text{FC } cif}\cdot \overbrace{c^{*,C}_{H,s}}^{\text{consumed}} + \overbrace{P^{C}_{F}}^{\text{HC } cif}\cdot \overbrace{c^{C}_{F,s}}^{\text{consumed}}}{P^{C}_{H}\cdot c^{C}_{H,s}+S^{C}_{s}\cdot \underbrace{P^{*,C}_{H}}_{\text{FC } cif}\cdot \underbrace{c^{*,C}_{H,s}}_{\text{consumed}}} =$$

$$= \frac{\overbrace{\frac{M_s}{M^*_s}}^{S^C_s} P^{*,C}_{H} \overbrace{\frac{1}{2}\left(\frac{P^{*,C}_{H}}{P^{*,C}}\right)^{-\nu}\frac{M^*_s}{P^{*,C}}}^{c^{*,C}_{H,s}} + P^{C}_{F}\overbrace{\frac{1}{2}\left(\frac{P^{C}_{F}}{P^{C}}\right)^{-\nu}\frac{M_s}{P^{C}}}^{c^{C}_{F,s}}}{P^{C}_{H}\underbrace{\frac{1}{2}\left(\frac{P^{C}_{H}}{P^{C}}\right)^{-\nu}\frac{M_s}{P^{C}}}_{c^{C}_{H,s}} + \underbrace{\frac{M_s}{M^*_s}}_{S^C_s} P^{*,C}_{H}\underbrace{\frac{1}{2}\left(\frac{P^{*,C}_{H}}{P^{*,C}}\right)^{-\nu}\frac{M^*_s}{P^{*,C}}}_{c^{*,C}_{H,s}}} =$$

Using our earlier result that, under CCP, *Home* and *Foreign* price levels are *equal*, due to the symmetry in the model, i.e. $P^{C} = P^{*,C}$, and dividing through by $\frac{M_s}{(P^C)^{1-\nu}}$, we obtain:

$$= \frac{P^{*,C}_{H}\left(P^{*,C}_{H}\right)^{-\nu} + P^{C}_{F}\left(P^{C}_{F}\right)^{-\nu}}{P^{C}_{H}\left(P^{C}_{H}\right)^{-\nu} + P^{*,C}_{H}\left(P^{*,C}_{H}\right)^{-\nu}} = \frac{(P^{*,C}_{H})^{1-\nu} + (P^{C}_{F})^{1-\nu}}{(P^{C}_{H})^{1-\nu} + (P^{*,C}_{H})^{1-\nu}} =$$

Recalling that $P_H^{*,C} = P_F^C$, due to the symmetry again, one can write:

$$= \frac{(P_H^{*,C})^{1-\nu} + (P_H^{*,C})^{1-\nu}}{(P_H^C)^{1-\nu} + (P_H^{*,C})^{1-\nu}} = \frac{2(P_H^{*,C})^{1-\nu}}{(P_H^C)^{1-\nu} + (P_H^{*,C})^{1-\nu}} =$$

Now dividing through by $(P_H^{*,C})^{1-\nu}$, we finally get:

$$= \frac{\dfrac{2(P_H^{*,C})^{1-\nu}}{(P_H^{*,C})^{1-\nu}}}{\dfrac{(P_H^C)^{1-\nu}}{(P_H^{*,C})^{1-\nu}} + \dfrac{(P_H^{*,C})^{1-\nu}}{(P_H^{*,C})^{1-\nu}}} = \frac{2}{\left(\dfrac{P_H^C}{P_H^{*,C}}\right)^{1-\nu} + 1}$$

So under CCP

$$(ft)_H^C = \frac{2}{\left(\frac{P_H^C}{P_H^{*,C}}\right)^{1-\nu} + 1} = \frac{2}{\left(\frac{E_0[u_{l,s} M_s]}{\frac{1}{1-\tau} E_0[u_{l,s} M_s^*]}\right)^{1-\nu} + 1} =$$

$$= \frac{2}{(1-\tau)^{1-\nu} + 1} = const \lesseqgtr 1 \text{ for } \nu \gtreqless 1,$$

which is (29) in the main text.
The derivation under PCP for Home is:

$$(ft)_{H,s}^P \equiv \frac{(FT)_{H,s}^P}{Y_{H,s}^P} = \frac{\overbrace{(Ex)_{H,s}^{P,fob}}^{\equiv (Ex)_{H,s}^{P,cif}} + \overbrace{(Im)_{H,s}^{P,fob}}^{\equiv (Im)_{H,s}^{P,cif}}}{(DA)_{H,s}^P + \underbrace{(Ex)_{H,s}^{P,fob}}_{\equiv (Ex)_{H,s}^{P,cif}}} = \frac{\overbrace{P_H^P}^{\text{HC } fob} \overbrace{\frac{c_{H,s}^{*,P}}{1-\tau}}^{\text{produced}} + S_s^P \overbrace{P_F^{*,P}}^{\text{FC } fob} \overbrace{\frac{c_{F,s}^P}{1-\tau}}^{\text{produced}}}{P_H^P c_{H,s}^P + \underbrace{P_H^P}_{\text{HC } fob} \underbrace{\frac{c_{H,s}^{*,P}}{1-\tau}}_{\text{produced}}} =$$

$$= \frac{\dfrac{P_H^P}{1-\tau} \overbrace{\dfrac{1}{2} \left(\dfrac{\dfrac{P_H^P}{S_s^P (1-\tau)}}{P_s^{*,P}}\right)^{-\nu} \dfrac{M_s^*}{P_s^{*,P}}}^{\equiv c_{H,s}^{*,P}} + \dfrac{S_s^P P_F^*}{1-\tau} \overbrace{\dfrac{1}{2} \left(\dfrac{\dfrac{S_s^P P_F^{*,P}}{1-\tau}}{P_s^P}\right)^{-\nu} \dfrac{M_s}{P_s^P}}^{\equiv c_{F,s}^P}}{P_H^P \underbrace{\dfrac{1}{2} \left(\dfrac{P_H^P}{P_s^P}\right)^{-\nu} \dfrac{M_s}{P_s^P}}_{\equiv c_H^P} + \dfrac{P_H^P}{1-\tau} \underbrace{\dfrac{1}{2} \left(\dfrac{\dfrac{P_H^P}{S_s^P (1-\tau)}}{P_s^{*,P}}\right)^{-\nu} \dfrac{M_s^*}{P_s^{*,P}}}_{\equiv c_{H,s}^{*,P}}} =$$

$$
= \frac{\frac{1}{2}\left(P_H^P\right)^{1-\nu}\left[\frac{1}{1-\tau}\left(\frac{\frac{1}{S_s^P\,(1-\tau)}}{P_s^{*,P}}\right)^{-\nu}\frac{M_s^*}{P_s^{*,P}}+\frac{S_s^P}{1-\tau}\left(\frac{\frac{S_s^P}{1-\tau}}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}\right]}{\frac{1}{2}\left(P_H^P\right)^{1-\nu}\left[\left(\frac{1}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}+\frac{1}{1-\tau}\left(\frac{\frac{1}{S_s^P\,(1-\tau)}}{P_s^{*,P}}\right)^{-\nu}\frac{M_s^*}{P_s^{*,P}}\right]} =
$$

$$
= \frac{\frac{1}{1-\tau}\left(\frac{\frac{1}{S_s^P\,(1-\tau)}}{P_s^{*,P}}\right)^{-\nu}\frac{M_s^*}{P_s^{*,P}}+\frac{S_s^P}{1-\tau}\left(\frac{\frac{S_s^P}{1-\tau}}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}}{\left(\frac{1}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}+\frac{1}{1-\tau}\left(\frac{\frac{1}{S_s^P\,(1-\tau)}}{P_s^{*,P}}\right)^{-\nu}\frac{M_s^*}{P_s^{*,P}}} =
$$

$$
= \frac{1+\frac{\frac{S_s^P}{1-\tau}\left(\frac{\frac{S_s^P}{1-\tau}}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}}{\left(\frac{1}{1-\tau}\right)^{1-\nu}\left(\frac{\frac{1}{S_s^P}}{P_s^{*,P}}\right)^{-\nu}\frac{M_s^*}{P_s^{*,P}}}}{\frac{\left(\frac{1}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}}{\left(\frac{1}{1-\tau}\right)^{1-\nu}\left(\frac{\frac{1}{S_s^P}}{P_s^{*,P}}\right)^{-\nu}\frac{M_s^*}{P_s^{*,P}}}+1} = \frac{1+S_s^P\frac{(1-\tau)^{\nu-1}\left(\frac{S_s^P}{P_s^P}\right)^{-\nu}}{(1-\tau)^{\nu-1}\left(\frac{\frac{1}{S_s^P}}{P_s^{*,P}}\right)^{-\nu}}\frac{M_s}{P_s^P}\frac{P_s^{*,P}}{M_s^*}}{\frac{\left(\frac{1}{P_s^P}\right)^{-\nu}}{(1-\tau)^{\nu-1}\left(\frac{\frac{1}{S_s^P}}{P_s^{*,P}}\right)^{-\nu}}\frac{M_s}{P_s^P}\frac{P_s^{*,P}}{M_s^*}+1} =
$$

$$
= \frac{1+S_s^P\left(\frac{S_s^P}{P_s^P}\frac{P_s^{*,P}}{\frac{1}{S_s^P}}\right)^{-\nu}\frac{M_s}{P_s^P}\frac{P_s^{*,P}}{M_s^*}}{(1-\tau)^{\nu-1}\left(\frac{1}{P_s^P}\frac{P_s^{*,P}}{\frac{1}{S_s^P}}\right)^{-\nu}\frac{M_s}{P_s^P}\frac{P_s^{*,P}}{M_s^*}+1} = \frac{1+S_s^P\left[\left(S_s^P\right)^2\frac{P_s^{*,P}}{P_s^P}\right]^{-\nu}\frac{M_s}{M_s^*}\frac{P_s^{*,P}}{P_s^P}}{\frac{1}{(1-\tau)^{\nu-1}}\left[S_s^P\frac{P_s^{*,P}}{P_s^P}\right]^{-\nu}\frac{M_s}{M_s^*}\frac{P_s^{*,P}}{P_s^P}+1} =
$$

$$= \frac{1 + \frac{S_s^P}{1-\tau}\left[\left(S_s^P\right)^2 \frac{P_s^{*,P}}{P_s^P}\right]^{-\nu} \frac{M_s}{M_s^*}\frac{P_s^{*,P}}{P_s^P}}{\left(\frac{1}{1-\tau}\right)^{\nu-1}\left[S_s^P \frac{P_s^{*,P}}{P_s^P}\right]^{-\nu} \frac{M_s}{M_s^*}\frac{P_s^{*,P}}{P_s^P} + 1} = \frac{1 + \left(S_s^P\right)^{1-2\nu} \frac{M_s}{M_s^*}\left(\frac{P_s^{*,P}}{P_s^P}\right)^{1-\nu}}{(1-\tau)^{1-\nu}\left(S_s^P\right)^{-\nu} \frac{M_s}{M_s^*}\left(\frac{P_s^{*,P}}{P_s^P}\right)^{1-\nu} + 1}.$$

Now recall our earlier result (33) that

$$S_s^P = \left(\frac{M_s^*}{M_s}\right)^{\frac{1}{1-2\nu}} \left(\frac{P_s^P}{P_s^{*,P}}\right)^{\frac{1-\nu}{1-2\nu}}.$$

Rearranging, we can write it as:

$$\left(S_s^P\right)^{1-2\nu} = \frac{M_s^*}{M_s}\left(\frac{P_s^P}{P_s^{*,P}}\right)^{1-\nu}.$$

Hence:

$$\frac{M_s}{M_s^*}\left(\frac{P_s^{*,P}}{P_s^P}\right)^{1-\nu} = \frac{1}{\left(S_s^P\right)^{1-2\nu}},$$

which we now use to substitute $\frac{M_s}{M_s^*}\left(\frac{P_s^{*,P}}{P_s^P}\right)^{1-\nu}$ out in our PCP Home trade share derivation, as we continue it below:

$$(ft)_{H,s}^P = \frac{1 + \left(S_s^P\right)^{1-2\nu} \frac{M_s}{M_s^*}\left(\frac{P_s^{*,P}}{P_s^P}\right)^{1-\nu}}{(1-\tau)^{1-\nu}\left(S_s^P\right)^{-\nu} \frac{M_s}{M_s^*}\left(\frac{P_s^{*,P}}{P_s^P}\right)^{1-\nu} + 1} =$$

$$= \frac{1 + \left(S_s^P\right)^{1-2\nu} \frac{1}{\left(S_s^P\right)^{1-2\nu}}}{(1-\tau)^{1-\nu}\left(S_s^P\right)^{-\nu} \frac{1}{\left(S_s^P\right)^{1-2\nu}} + 1} =$$

$$= \frac{1+1}{(1-\tau)^{1-\nu}\left(S_s^P\right)^{-\nu-1+2\nu} + 1} =$$

$$= \frac{2}{(1-\tau)^{1-\nu}\left(S_s^P\right)^{\nu-1} + 1}$$

We can finally write:

$$(ft)_{H,s}^P = \frac{2}{(1-\tau)^{1-\nu}\left(S_s^P\right)^{\nu-1} + 1} =$$

$$= \frac{2}{(1-\tau)^{1-\nu} \underbrace{\left\{ \left(\frac{M_s^*}{M_s}\right)^{\frac{1}{1-2\nu}} \left(\frac{P_s^P}{P_s^{*,P}}\right)^{\frac{1-\nu}{1-2\nu}} \right\}}_{=S_s^P}^{\nu-1} + 1} \neq const \text{ unless } S_s^P = 1,$$

which is (30) in the main text.
The respective expressions for Foreign can be derived by analogy to be:

$$(ft)_{F,s}^C = \frac{2}{\left(\frac{P_F^{*,C}}{P_H^C}\right)^{1-\nu} + 1} = \frac{2}{\left(\frac{E_0[u_l^* M_s^*]}{\frac{1}{1-\tau} E_0[u_l^* M_s^*]}\right)^{1-\nu} + 1} =$$

$$= \frac{2}{(1-\tau)^{1-\nu} + 1} = const \lesseqgtr 1 \text{ for } \nu \gtreqless 1 \text{ vs.}$$

$$(ft)_{F,s}^P = \frac{2}{(1-\tau)^{1-\nu} \underbrace{\left[\left(\frac{M_s^*}{M_s}\right)^{\frac{1}{1-2\nu}} \left(\frac{P_s^P}{P_s^{*,P}}\right)^{\frac{1-\nu}{1-2\nu}} \right]}_{=S_s^P}^{1-\nu} + 1} \neq const \text{ unless } S_s^P = 1,$$

which correspond to (31) and (32) in the main text.

B Proofs of Propositions

B.1 Proposition 1 (Relative Consumption)

Proof.

- Under *CCP* and *full* symmetry with *separable* preferences and *iceberg* costs (recall that in this case $P_H^C = P_F^{*,C}$ and $P_H^{*,C} = P_F^C$ and thus $P^C = P^{*,C}$), *relative* real consumption can be expressed as follows:

$$\frac{c_s^C}{c_s^{*,C}} \equiv \frac{c_{H,s}^C + c_{F,s}^C}{c_{F,s}^{*,C} + c_{H,s}^{*,C}} = \frac{\frac{1}{2}\left(\frac{P_H^C}{P^C}\right)^{-\nu}\frac{M_s}{P^C} + \frac{1}{2}\left(\frac{P_F^C}{P^C}\right)^{-\nu}\frac{M_s}{P^C}}{\frac{1}{2}\left(\frac{P_F^{*,C}}{P^{*,C}}\right)^{-\nu}\frac{M_s^*}{P^{*,C}} + \frac{1}{2}\left(\frac{P_H^{*,C}}{P^{*,C}}\right)^{-\nu}\frac{M_s^*}{P^{*,C}}} =$$

$$= \frac{\frac{1}{2}\left(\frac{1}{P^C}\right)^{-\nu}\frac{M_s}{P^C}\left[\left(P_H^C\right)^{-\nu} + \left(P_F^C\right)^{-\nu}\right]}{\frac{1}{2}\left(\frac{1}{P^{*,C}}\right)^{-\nu}\frac{M_s^*}{P^{*,C}}\left[\left(P_F^{*,C}\right)^{-\nu} + \left(P_H^{*,C}\right)^{-\nu}\right]} = \frac{M_s}{M_s^*} = S_s^C \neq 1 \text{ unless } M_s = M_s^*;$$

- Under *PCP* and *full* symmetry with *separable* preferences and *iceberg* costs (so that $P_H^P = P_F^{*,P}$, $P_{F,s}^P = \frac{S_s^P P_F^{*,P}}{1-\tau}$, $P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P(1-\tau)}$ and thus $P_s^P = \left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}} P_s^{*,P}$), analogous reasoning derives *relative* real consumption to be:

$$\frac{c_s^P}{c_s^{*,P}} \equiv \frac{c_{H,s}^P + c_{F,s}^P}{c_{F,s}^{*,P} + c_{H,s}^{*,P}} = \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P} + \frac{1}{2}\left(\frac{P_{F,s}^P}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}}{\frac{1}{2}\left(\frac{P_F^{*,P}}{P_s^{*,P}}\right)^{-\nu}\frac{M_s^*}{P_s^{*,P}} + \frac{1}{2}\left(\frac{P_{H,s}^{*,P}}{P_s^{*,P}}\right)^{-\nu}\frac{M_s^*}{P_s^{*,P}}} =$$

$$= \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P} + \frac{1}{2}\left(\frac{\frac{S_s^P P_F^{*,P}}{1-\tau}}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}}{\frac{1}{2}\left(\frac{\frac{P_F^{*,P}}{P_s^P}}{\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}}\right)^{-\nu}\frac{\frac{M_s^*}{P_s^P}}{\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}} + \frac{1}{2}\left(\frac{\frac{\frac{P_H^P}{S_s^P(1-\tau)}}{P_s^P}}{\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}}\right)^{-\nu}\frac{\frac{M_s^*}{P_s^P}}{\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}}} =$$

$$
= \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}\left[1+\left(\frac{\frac{S_s^P}{1-\tau}}{1}\right)^{-\nu}\right]}{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu}\frac{M_s^*}{P_s^P}\left(\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}\right)^{1-\nu}+\left(\frac{1}{S_s^P(1-\tau)}\right)^{-\nu}\left(\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}\right)^{1-\nu}} =
$$

$$
= \frac{M_s}{M_s^*}\frac{\left[1+\left(\frac{S_s^P}{1-\tau}\right)^{-\nu}\right]}{\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}+\left(\frac{1}{S_s^P(1-\tau)}\right)^{-\nu}\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}} =
$$

$$
= \frac{M_s}{M_s^*}\frac{\left[1+\left(\frac{S_s^P}{1-\tau}\right)^{-\nu}\right]}{\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\left[1+\left(\frac{1}{S_s^P(1-\tau)}\right)^{-\nu}\right]} \neq 1 \text{ unless } M_s = M_s^* \Rightarrow S_s^P = 1.
$$

We finally derive the above formula for two special cases widely exploited in the NOEM literature, namely $\tau = 0$ and $\nu = 1$.

For $\tau = 0$:

$$
\frac{c_s^P}{c_s^{*,P}} = \frac{M_s}{M_s^*}\frac{\left[1+\left(\frac{S_s^P}{1-0}\right)^{-\nu}\right]}{\frac{1+\left(\frac{1}{1-0}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-0}\frac{1}{S_s^P}\right)^{1-\nu}}\left[1+\left(\frac{1}{S_s^P(1-0)}\right)^{-\nu}\right]} =
$$

$$
= \frac{M_s}{M_s^*}\frac{\left[1+\left(S_s^P\right)^{-\nu}\right]}{\frac{1+(S_s^P)^{1-\nu}}{1+\left(\frac{1}{S_s^P}\right)^{1-\nu}}\left[1+\left(\frac{1}{S_s^P}\right)^{-\nu}\right]} =
$$

$$
= \frac{M_s}{M_s^*}\frac{\left[1+\left(S_s^P\right)^{-\nu}\right]\left[1+\left(S_s^P\right)^{\nu-1}\right]}{\left[1+\left(S_s^P\right)^{1-\nu}\right]\left[1+\left(S_s^P\right)^{\nu}\right]} =
$$

$$
= \frac{M_s}{M_s^*}\frac{1+\left(S_s^P\right)^{\nu-1}+\left(S_s^P\right)^{-\nu}+S_s^P}{1+\left(S_s^P\right)^{\nu}+\left(S_s^P\right)^{1-\nu}+S_s^P} =
$$

$$
= \frac{M_s}{M_s^*}\frac{1+S_s^P+\left(\frac{1}{S_s^P}\right)^{\nu}+\left(\frac{1}{S_s^P}\right)^{1-\nu}}{1+S_s^P+\left(S_s^P\right)^{\nu}+\left(S_s^P\right)^{1-\nu}} \neq 1 \text{ unless } M_s = M_s^* \Rightarrow S_s^P = 1 \text{ with } 0 < \nu << \infty \Rightarrow
$$

$$\Rightarrow \frac{c_s^P}{c_s^{*,P}} = 1 \text{ with } \nu \to \infty \text{ when } \tau = 0.$$

For $\nu = 1$:

$$\frac{c_s^P}{c_s^{*,P}} = \frac{M_s}{M_s^*} \frac{\left[1 + \left(\frac{S_s^P}{1-\tau}\right)^{-1}\right]}{\frac{1+\left(\frac{1}{1-\tau} S_s^P\right)^{1-1}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-1}} \left[1 + \left(\frac{1}{S_s^P(1-\tau)}\right)^{-1}\right]} =$$

$$= \frac{M_s}{M_s^*} \frac{1 + \frac{1-\tau}{S_s^P}}{\frac{1+1}{1+1}\left[1 + S_s^P (1-\tau)\right]} =$$

$$= \frac{M_s}{M_s^*} \frac{1 + \frac{1-\tau}{S_s^P}}{1 + S_s^P (1-\tau)} \neq 1 \text{ unless } M_s = M_s^* \Rightarrow S_s^P = 1 \text{ with } 0 \leq \tau \leq 1.$$

This completes our proof. ∎

B.2 Proposition 2 (Home Bias)

Proof.

- Under *CCP* and *full* symmetry with *separable* preferences and *iceberg* costs ($P_H^C = P_F^{*,C}$ and $P_H^{*,C} = P_F^C$ and thus $P^C = P^{*,C}$), the optimal *split-up* of real consumption between demand for domestic and foreign goods can be expressed as follows.

For *Home*:

$$\frac{c_{H,s}^C}{c_{F,s}^C} = \frac{\frac{1}{2}\left(\frac{P_H^C}{P^C}\right)^{-\nu} \frac{M_s}{P^C}}{\frac{1}{2}\left(\frac{P_F^C}{P^C}\right)^{-\nu} \frac{M_s}{P^C}} = \frac{\left(\frac{\varphi}{\varphi-1} P^C \frac{E_0[u_{l,s} M_s]}{E_0[u_{c,s} M_s]}\right)^{-\nu}}{\left(\frac{1}{1-\tau}\frac{\varphi}{\varphi-1} P^{,C} \frac{E_0\left[u_{l,s}^* M_s\right]}{E_0\left[u_{c,s}^* M_s^*\right]}\right)^{-\nu}} =$$

$$= \frac{1}{\left(\frac{1}{1-\tau}\right)^{-\nu}} = (1-\tau)^{-\nu} > 1, \text{ for } \forall s \in S \text{ when } 0 < \tau < 1 \text{ and } \nu > 0 \Rightarrow$$

$$\Rightarrow \frac{c_{H,s}^P}{c_{F,s}^P} = 1 \text{ if (1) } \tau = 0, \text{ for } \forall s \in S, \text{ or (2) } \nu = 0, \text{ for } \forall s \in S.$$

For *Foreign*:

$$\frac{c_{F,s}^{*,C}}{c_{H,s}^{*,C}} = \frac{\frac{1}{2}\left(\frac{P_F^{*,C}}{P^{*,C}}\right)^{-\nu} \frac{M_s^*}{P^{*,C}}}{\frac{1}{2}\left(\frac{P_H^{*,C}}{P^{*,C}}\right)^{-\nu} \frac{M_s^*}{P^{*,C}}} = \frac{\left(\frac{\varphi}{\varphi-1} P^{*,C} \frac{E_0\left[u_{l,s}^* M_s^*\right]}{E_0\left[u_{c,s}^* M_s^*\right]}\right)^{-\nu}}{\left(\frac{1}{1-\tau}\frac{\varphi}{\varphi-1} P^{*,C} \frac{E_0[u_{l,s} M_s^*]}{E_0[u_{c,s} M_s]}\right)^{-\nu}} =$$

$$= \frac{1}{\left(\frac{1}{1-\tau}\right)^{-\nu}} = (1-\tau)^{-\nu} > 1, \text{ for } \forall s \in S \text{ when } 0 < \tau < 1 \text{ and } \nu > 0 \Rightarrow$$

$$\Rightarrow \frac{c_{H,s}^{P}}{c_{F,s}^{P}} = 1 \text{ if (1) } \tau = 0, \text{ for } \forall s \in S, \text{ or (2) } \nu = 0, \text{ for } \forall s \in S.$$

- Under *PCP* and *full* symmetry with *separable* preferences and *iceberg* costs ($P_H^P = P_F^{*,P}$, $P_{F,s}^P = S_s^P \frac{P_F^{*,P}}{1-\tau}$, $P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P(1-\tau)}$ and thus $P_s^P = \left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}} P_s^{*,P}$), analogous reasoning derives the optimal *split-up* of real consumption, as follows.

For *Home*:

$$\frac{c_{H,s}^{P}}{c_{F,s}^{P}} = \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}}{\frac{1}{2}\left(\frac{\frac{S_s^P P_F^{*,P}}{1-\tau}}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}} = \frac{1}{\left(\frac{S_s^P}{1-\tau}\right)^{-\nu}} = \left(\frac{1-\tau}{S_s^P}\right)^{-\nu} = \frac{(1-\tau)^{-\nu}}{\left(S_s^P\right)^{-\nu}} =$$

$$= \underbrace{\left(S_s^P\right)^{\nu}\overbrace{(1-\tau)^{-\nu}}^{\equiv CCP \text{ home bias}}}_{\equiv PCP \text{ optimal split-up in } Home} \lesseqgtr 1 \text{ for } S_s^P \lesseqgtr 1-\tau \text{ when } 0 < \tau < 1 \text{ and } \nu > 0 \Rightarrow$$

$$\Rightarrow \frac{c_{H,s}^{P}}{c_{F,s}^{P}} = 1 \text{ if (1) } S_{s_\tau}^P = 1-\tau, \text{ for } \forall s_\tau \subset S \text{ or (2) } \nu = 0, \text{ for } \forall s \in S.$$

For *Foreign*:

$$\frac{c_{F,s}^{*,C}}{c_{H,s}^{*,C}} = \frac{\frac{1}{2}\left(\frac{P_F^{*,P}}{P_s^{*,P}}\right)^{-\nu}\frac{M_s^*}{P_s^{*,P}}}{\frac{1}{2}\left[\frac{\frac{P_H^P}{S_s^P(1-\tau)}}{P_s^{*,P}}\right]^{-\nu}\frac{M_s^*}{P_s^{*,P}}} = \frac{1}{\left[\frac{1}{S_s^P(1-\tau)}\right]^{-\nu}} = \left[S_s^P(1-\tau)\right]^{-\nu} =$$

$$= \underbrace{\left(S_s^P\right)^{-\nu}\overbrace{(1-\tau)^{-\nu}}^{\equiv CCP \text{ home bias}}}_{\equiv PCP \text{ optimal split-up in } Foreign} \lesseqgtr 1 \text{ for } \frac{1}{S_s^P} \gtreqless 1-\tau, \text{ when } 0 < \tau < 1 \text{ and } \nu > 0 \Rightarrow$$

$$\Rightarrow \frac{c_{H,s}^{P}}{c_{F,s}^{P}} = 1 \text{ if (1) } S_{s_\tau}^P = 1-\tau, \text{ for } \forall s_\tau \subset S \text{ or (2) } \nu = 0, \text{ for } \forall s \in S.$$

This completes our proof. ■

B.3 Proposition 3 (Relative Leisure)

Proof.

- Under *CCP* and *full* symmetry with *separable* preferences and *iceberg* costs ($P_H^C = P_F^{*,C}$, $P_H^{*,C} = P_F^C$ and $P^C = P^{*,C}$), *relative* real output can be expressed as:

$$\frac{y_s^C}{y_s^{*,C}} \equiv \frac{c_{H,s}^C + c_{H,s}^{*,C}}{c_{F,s}^{*,C} + c_{F,s}^C} = \frac{\frac{1}{2}\left(\frac{P_H^C}{P^C}\right)^{-\nu} \frac{M_s}{P^C} + \frac{1}{2}\left(\frac{P_H^{*,C}}{P^{*,C}}\right)^{-\nu} \frac{M_s^*}{P^{*,C}}}{\frac{1}{2}\left(\frac{P_F^{*,C}}{P^{*,C}}\right)^{-\nu} \frac{M_s^*}{P^{*,C}} + \frac{1}{2}\left(\frac{P_F^C}{P^C}\right)^{-\nu} \frac{M_s}{P^C}} =$$

$$= \frac{\frac{1}{2}\left(\frac{P_H^C}{P^C}\right)^{-\nu} \frac{1}{P^C}[M_s + (1-\tau)^\nu M_s^*]}{\frac{1}{2}\left(\frac{P_F^{*,C}}{P^{*,C}}\right)^{-\nu} \frac{1}{P^{*,C}}[M_s^* + (1-\tau)^\nu M_s]} = \frac{M_s + (1-\tau)^\nu M_s^*}{M_s^* + (1-\tau)^\nu M_s} \neq 1 \text{ unless } M_s = M_s^* \text{ or } \tau = 0 \text{ or } \nu = 0 \Leftrightarrow$$

$$\Leftrightarrow y_s^C \neq y_s^{*,C} \text{ unless } M_s = M_s^* \text{ or } \tau = 0 \text{ or } \nu = 0.$$

Since $\frac{y_s^C}{y_s^{*,C}} \equiv \frac{n_s^C}{n_s^{*,C}} \equiv \frac{1-l_s^C}{1-l_s^{*,C}} \neq 1$ (unless $M_s = M_s^*$ or $\tau = 0$ or $\nu = 0$):

$$\frac{l_s^C}{l_s^{*,C}} \neq 1 \Leftrightarrow l_s^C \neq l_s^{*,C} \text{ unless } M_s = M_s^* \text{ or } \tau = 0 \text{ or } \nu = 0.$$

- Under *PCP* and *full* symmetry with *separable* preferences and *iceberg* costs ($P_H^P = P_F^{*,P}$, $P_{F,s}^P = \frac{S_s^P P_F^{*,P}}{1-\tau}$, $P_{H,s}^{*,P} = \frac{P_H^P}{S_s^P(1-\tau)}$ and $P_s^P = \left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}} P_s^{*,P}$), analogous reasoning derives *relative* real output to be:

$$\frac{y^P}{y^{*,P}} \equiv \frac{c_{H,s}^P + c_{H,s}^{*,P}}{c_{F,s}^{*,P} + c_{F,s}^P} = \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu} \frac{M_s}{P_s^P} + \frac{1}{2}\left(\frac{P_{H,s}^{*,P}}{P_s^{*,P}}\right)^{-\nu} \frac{M_s^*}{P_s^{*,P}}}{\frac{1}{2}\left(\frac{P_F^{*,P}}{P_s^{*,P}}\right)^{-\nu} \frac{M_s^*}{P_s^{*,P}} + \frac{1}{2}\left(\frac{P_{F,s}^P}{P_s^P}\right)^{-\nu} \frac{M_s}{P_s^P}} =$$

$$
= \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}+\frac{1}{2}\left(\frac{\frac{\frac{P_H^P}{S_s^P(1-\tau)}}{P_s^P}}{\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}}\right)^{-\nu}\frac{\frac{M_s^*}{P_s^P}}{\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}}}{\frac{1}{2}\left(\frac{\frac{P_F^{*,P}}{P_s^P}}{\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}}\right)^{-\nu}\frac{\frac{M_s^*}{P_s^P}}{\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}}+\frac{1}{2}\left(\frac{\frac{S_s^P P_F^{*,P}}{1-\tau}}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}} =
$$

$$
= \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}+\frac{1}{2}\left(\frac{\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}\frac{P_H^P}{S_s^P(1-\tau)}}{P_s^P}\right)^{-\nu}\frac{\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}M_s^*}{P_s^P}}{\frac{1}{2}\left(\frac{\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}P_F^{*,P}}{P_s^P}\right)^{-\nu}\frac{\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}M_s^*}{P_s^P}+\frac{1}{2}\left(\frac{\frac{S_s^P P_F^{*,P}}{1-\tau}}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}} =
$$

$$
= \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu}\frac{M_s}{P_s^P}+\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu}\left(\frac{1}{S^P(1-\tau)}\right)^{-\nu}\frac{\left(\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}\right)^{1-\nu}M_s^*}{P_s^P}}{\frac{1}{2}\left(\frac{P_F^{*,P}}{P_s^P}\right)^{-\nu}\frac{\left(\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}\right]^{\frac{1}{1-\nu}}\right)^{1-\nu}M_s^*}{P_s^P}+\frac{1}{2}\left(\frac{P_F^{*,P}}{P_s^P}\right)^{-\nu}\left(\frac{S_s^P}{1-\tau}\right)^{-\nu}\frac{M_s}{P_s^P}} =
$$

$$
= \frac{\frac{1}{2}\left(\frac{P_H^P}{P_s^P}\right)^{-\nu}\frac{1}{P_s^P}\left[M_s+\left(\frac{1}{S_s^P(1-\tau)}\right)^{-\nu}\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}M_s^*\right]}{\frac{1}{2}\left(\frac{P_F^{*,P}}{P_s^P}\right)^{-\nu}\frac{1}{P_s^P}\left[\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}}M_s^*+\left(\frac{S_s^P}{1-\tau}\right)^{-\nu}M_s\right]} =
$$

$$= \frac{M_s + \left(\frac{1}{S_s^P(1-\tau)}\right)^{-\nu} \frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}} M_s^*}{\frac{1+\left(\frac{1}{1-\tau}S_s^P\right)^{1-\nu}}{1+\left(\frac{1}{1-\tau}\frac{1}{S_s^P}\right)^{1-\nu}} M_s^* + \left(\frac{S_s^P}{1-\tau}\right)^{-\nu} M_s} \neq 1 \text{ unless } M_s = M_s^* \Rightarrow S_s^P = 1.$$

Since $\frac{y_s^P}{y_s^{*,P}} \equiv \frac{n_s^P}{n_s^{*,P}} \equiv \frac{1-l_s^P}{1-l_s^{*,P}} \neq 1$ unless $M_s = M_s^* \Rightarrow S_s^P = 1$, then:

$$\frac{l_s^P}{l_s^{*,P}} \neq 1 \Leftrightarrow l_s^P \neq l_s^{*,P} \text{unless } M_s = M_s^* \Rightarrow S_s^P = 1.$$

This completes our proof. ■

B.4 Proposition (Expected Trade Shares under PCP)

Proof. [38] Recall the formulas for the equilibrium trade shares we derived for Home, (30), and Foreign, (32), under PCP (skipping the P supersript for convenience since now, in the PCP case, there is no ambiguity on invoicing):

$$ft_H(S_s) = \frac{2}{(1-\tau)^{1-\nu} S_s^{\nu-1} + 1} \qquad \text{and} \qquad ft_F(S_s) = \frac{2}{(1-\tau)^{1-\nu} S_s^{1-\nu} + 1}.$$

With full symmetry, as assumed:

$$ft_H(S_s) = ft_F(1/S_s).$$

Let us first focus on Home's trade share.

Symmetry in our particular context here implies that for each state of nature s there is a symmetric state s' such that:

1. the exchange rate is inverse: $S_{s'} = 1/S_s$;
2. the two states have the same probability: $\pi_s = \pi_{s'}$.

Then expected trade share *across the two symmetric states in the pair* is:

$$\begin{aligned}
E_0\left[ft_{H,(s,s')}(S_s)\right] &= \pi_s ft_H(S_s) + \pi_s ft_H(S_{s'}) = \\
&= \pi_s \frac{2}{(1-\tau)^{1-\nu} S_s^{\nu-1} + 1} + \pi_s \frac{2}{(1-\tau)^{1-\nu} S_s^{1-\nu} + 1} = \\
&= \pi_s \frac{2}{(1-\tau)^{1-\nu} + 1}\left[\frac{(1-\tau)^{1-\nu}+1}{(1-\tau)^{1-\nu} S_s^{\nu-1} + 1} + \frac{(1-\tau)^{1-\nu}+1}{(1-\tau)^{1-\nu} S_s^{1-\nu} + 1}\right] = \\
&= \pi_s ft_{H,peg} F(S_s).
\end{aligned}$$

[38] I am indebted to Cédric Tille for suggesting to me this straightforward proof.

The expectation is thus equal to the constant trade share under peg (or CCP), $ft_{H,peg} = \frac{2}{(1-\tau)^{1-\nu}+1}$, times the function $F(S_s) = \frac{(1-\tau)^{1-\nu}+1}{(1-\tau)^{1-\nu}S_s^{\nu-1}+1} + \frac{(1-\tau)^{1-\nu}+1}{(1-\tau)^{1-\nu}S_s^{1-\nu}+1}$.

For a benchmark, consider what would be the value of the above expectation if the trade share was a constant, i.e. $ft_H(S_s) = ft_H(S_{s'}) = ft_{H,peg}$. Then one would have:

$$E_0\left[ft_{H,(s,s')}(S_s)\right] = 2\pi_s ft_{H,peg}$$

We shall now show that $F(S_s) \geq 2$, which would mean that the expected trade share for each *pair of symmetric states* exceeds the corresponding trade share under peg.

One can easily prove that:

$$F(S_s) = F(1/S_s) \qquad \text{and} \qquad F(1) = 2,$$

i.e. that $F(S_s)$ is a symmetric function equal to 2 when the exchange rate is 1. We further write:

$$\begin{aligned} F'(S_s) &= (\nu-1)(1-\tau)^{1-\nu}\left[(1-\tau)^{1-\nu}+1\right] \times \\ &\times \left(-\frac{S_s^{\nu-2}}{\left[(1-\tau)^{1-\nu}S_s^{\nu-1}+1\right]^2} + \frac{S_s^{-\nu}}{\left[(1-\tau)^{1-\nu}S_s^{1-\nu}+1\right]^2} \right), \\ F'(1) &= 0, \end{aligned}$$

so that $F(S_s)$ is flat at $S=1$. Moreover:

$$\begin{aligned} F''(S_s) &= (\nu-1)(1-\tau)^{1-\nu}\left[(1-\tau)^{1-\nu}+1\right] \times \\ &\times \left[\begin{array}{c} -(\nu-2)\frac{S_s^{\nu-3}}{\left[(1-\tau)^{1-\nu}S_s^{\nu-1}+1\right]^2} + 2(\nu-1)\frac{S_s^{\nu-2}(1-\tau)^{1-\nu}S_s^{\nu-2}}{\left[(1-\tau)^{1-\nu}S_s^{\nu-1}+1\right]^3} \\ -\nu\frac{S_s^{-\nu-1}}{\left[(1-\tau)^{1-\nu}S_s^{1-\nu}+1\right]^2} + 2(\nu-1)\frac{(1-\tau)^{1-\nu}S_s^{-\nu}S_s^{-\nu}}{\left[(1-\tau)^{1-\nu}S_s^{1-\nu}+1\right]^3} \end{array} \right], \\ F''(1) &= \frac{2(\nu-1)^2(1-\tau)^{2(1-\nu)}}{\left[(1-\tau)^{1-\nu}+1\right]^2}\left[1-(1-\tau)^{\nu-1}\right]. \end{aligned}$$

We now have to consider two cases, in addition to the trivial third case of unit substitutability when the trade share is constant at 1.

- *Elastic* import demand, $\nu > 1$. In this case $F(S_s)$ is *convex* around $S=1$, which proves that the function $F(S_s)$ attains a local *minimum* $F(1) = 2$ around $S=1$. Then it follows that $F(S_s) \geq 2$, at least around $S=1$ (the region in which we are interested in, particularly under price rigidity compatible with relatively small money shocks as assumed in this paper). Finally, summing over *all* pairs of symmetric states, we obtain:

$$E_0\left[ft_H(S_s)\right] \geq ft_{H,peg} \Leftrightarrow F(S_s) \geq 2.$$

The same arguments apply for Foreign's expected trade share. Adding up the expected trade share for the two countries in the model leads to the conclusion that expected trade in terms of output is *(slightly) lower under peg* than under float, trade costs and import demand elasticity.

- *Inelastic* import demand, $0 < \nu < 1$. In this case $F(S_s)$ is *concave* around $S = 1$, which proves that the function $F(S_s)$ attains a local *maximum* $F(1) = 2$ around $S = 1$. Then it follows that $F(S_s) \leq 2$, at least around $S = 1$ (the region in which we are interested in, particularly under price rigidity compatible with relatively small money shocks as assumed in this paper). Finally, summing over *all* pairs of symmetric states, we obtain:

$$E_0 \left[ft_H \left(S_s \right) \right] \leq ft_{H,peg} \Leftrightarrow F \left(S_s \right) \leq 2.$$

  The same arguments apply for Foreign's expected trade share. Adding up the expected trade share for the two countries in the model leads to the conclusion that expected trade in terms of output is *(slightly) higher under peg* than under float, trade costs and import demand inelasticity.

This completes our proof. ■

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